A Theory of Vote Trading and Information Aggregation

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Abstract. The ability to buy “empty votes” by using derivative contracts to separate shares’ voting and cash flow rights is a cause of policy concern. We construct a formal model of a market for empty votes where informed and uninformed (and potentially biased) shareholders endogenously choose to demand or supply. There always exists an equilibrium where only informed shareholders demand votes, since uninformed shareholders’ willingness to pay for votes is endogenously reduced by two effects: the “swing bidders’ curse” and the “crowding out effect”. Votes are transferred from uninformed to informed actors, thus trading improves welfare by better aggregating information. We provide conditions under which trading can achieve full information aggregation. In the presence of frictions such as forced voting or contrarians (short sellers) the endogeneity of vote supply can limit the damage they can cause: if they are too prevalent then uninformed and unbiased shareholders simply refrain from supplying.

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1. Introduction

The idea of one-share-one-vote has long been enshrined as an important principle not only of liberal democracy, but also of corporate charters. Corporations departing from this norm are generally regarded as departing from good governance (Gompers et al. 2003). And yet the existence of markets for lending securities as well as of derivative markets, where the economic risk associated with share purchases can be laid off, makes it relatively easy to acquire votes without the associated cash flow rights. That is, under current regulations, votes can be bought or borrowed separately from shares. The fact that investors with conflicts of interest can easily acquire votes in corporate elections has led legal commentators to express concern about the occurrence of “empty voting,” and to consider how it might best be regulated (Hu and Black 2006, 2007). Their arguments are intuitive; in this paper we take some first steps in bringing formal economic analysis to bear on this question.

Consider a voting model in which voters may be informed or uninformed about the “correct” way to vote (i.e., some voters know which alternative will maximize corporate value, whereas others do not). Such a situation arises naturally in corporate voting contexts where some shareholders may lack the time, resources, or incentives to determine whether (for example) a merger creates or destroys value. There is no benefit to voting unless one’s vote changes the decision—and yet uninformed shareholders do not want their votes to swing the election decision when they are voting against the choices of informed voters. This effect is known as the “swing voter’s curse” (Feddersen and Pessendorfer 1996). Depending on the fraction of informed and uninformed voters among the shareholders of a corporation, this could lead to relatively low rates of turnout. Under these circumstances, uninformed shareholders may even be willing to lend out their votes in order to enhance the voting power of informed shareholders.

Christoffersen et al. (2007) find evidence for such behavior in their empirical study of the share-lending market. They find spikes in the volume of share lending around record dates. Note that when a share is borrowed, there is no exposure to the cash-flow rights of the share as the contract usually specifies that any dividends must be returned to the lender. However, the voting rights associated with the share belong to the borrower for the duration of the loan. Interestingly, Christoffersen et al. (2007) do not see a corresponding spike in the prices of most stock loans around record dates: the average incremental price of borrowing a share in order to vote it is zero. Therefore they suggest that vote-trading may be beneficial: stock lenders around record dates are happy to allow other — perhaps more informed — agents to borrow their shares in order to vote them. However,

1The average turnout at shareholder votes varies widely across different countries and is documented in Hewitt (2011). See also the discussion below for the special rules affecting abstention in corporate elections which affect voter turnout in the US.

2The record date is the date on which a shareholder must own a share in order to be eligible to vote it in any upcoming corporate election.
the dark side of vote-trading, as Hu and Black (2006, 2007) point out, is that some voters may have a “bias:” an idiosyncratic preference over the alternatives they are asked to vote on. Share-lending and vote-trading might allow these biased actors to acquire extra voting power without any corresponding change in their economic interest, and so they will have a greater ability to sway the election from the welfare-maximizing choice in the direction of their partisan bias. The theoretical literature has not yet tackled the question of whether uninformed, unbiased individuals would be willing to supply their votes in a market where the demanders of votes may be either biased or uninformed; nor whether such trades, if they occur, are likely to be welfare-improving. This paper aims to take the first step in answering this important question.

We begin by setting out and solving a baseline voting model without vote-trading. Each shareholder-voter — either informed or uninformed of the socially optimal decision — receives both a common- and a private-value payoff component, the latter corresponding to the shareholder’s idiosyncratic “partisan” preference for the outcome of the vote. A fixed number of informed shareholders vote for the socially optimal decision, but there are also votes cast at random (e.g., by noise voters or sufficiently strong partisans). We show that in the unique equilibrium, sufficiently unbiased uninformed shareholders abstain (to avoid the swing voter’s curse); however, with sufficiently great bias, they vote for their preferred outcome. This is a game of strategic complementarities in that uninformed partisans receive a higher benefit from abstaining if they expect others to do so (because it becomes more likely that their votes are pivotal against informed agents’ votes). Social welfare is higher with more abstention by uninformed agents, which would result from having more informed shareholders (relative to uninformed ones), or stochastically less spread-out idiosyncratic preferences among uninformed shareholders.

We then introduce the possibility for shareholders to sell their votes (without their shares) to other agents in a frictionless vote-trading market. To our knowledge this is the first model of vote trading where some participants have information about the correct choice and both the supply of and the demand for votes are endogenous. We show that there is an equilibrium such that the price of an empty vote is zero, consistent with the empirical

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3Hu and Black (2006, 2007) refer to agents who hold voting power disproportionately to their economic interest as “empty voters”. Their main concern about the separation of voting and economic ownership is that empty voters are often biased or conflicted voters — in the sense of this paper. An empty voter has an incentive to destroy value in a corporation when biased by, say, an offsetting derivative position, or a position in a related company, etc.

4The election is modeled as a Poisson game, an approach pioneered by Myerson (1998a,b); our model is most closely related to that of Feddersen and Pesendorfer (1999).

5In a corporate setting, management may be biased against a merger bringing uncertain benefits to shareholders at the cost of job losses, or alternatively favor an acquisition that might be value-decreasing for empire-building reasons. Another example of a partisan voter might be a hedge fund which votes the shares of an acquirer in favor of a takeover damaging current shareholders because the transaction will be good for another company (e.g. the target) which is in the hedge fund’s portfolio. Strong partisans can be thought of as “conflicted” voters who will not necessarily vote to maximize the shareholder value.

6Related papers on vote-trading are discussed below.
observations of Christoffersen et al. (2007). All uninformed, strategic shareholders (who by assumption would prefer to induce the socially optimal decision were they informed what it was) supply their votes at zero price, hoping that their votes will be picked up by informed voters. (As a result, more votes are supplied at zero price than the number that would have been abstained without a vote market.) In equilibrium, all supplied votes are picked up and optimally cast by informed shareholders, which improves social welfare. An indirect effect (either off the equilibrium path, or on-path in case of frictions) is that vote trading encourages more abstention by uninformed shareholders who happen to retain their votes at the record date. This is so because vote trading increases the proportion of informed voters and hence the cost of uninformed voting due to the swing voter’s curse.

Finally, we consider the effects of exogenous frictions — such as supplied votes being returned, or diverted to be cast at random, or even cast specifically against the socially-optimal decision by contrarians — on the equilibrium supply decision of uninformed and perfectly unbiased shareholders in the zero-price equilibrium. Such frictions may render vote trading socially undesirable. However, we show that uninformed, perfectly unbiased shareholders (aware of the frictions) choose not to supply their votes in the vote market exactly when trading at zero price is socially detrimental. In this sense, the vote market is self-regulating provided the price of an empty vote remains zero either endogenously or by regulatory intervention.

Our results concerning the efficiency of vote trading (and when inefficient, the collapse of the market for empty votes at zero supply price) should be viewed as a benchmark. The model and methodology proposed in this paper can be used to evaluate policy proposals when this benchmark is not achieved. For instance, the model highlights the importance of transparent and voluntary supply of empty votes, preferably at a negligible price, so that the vote supply decisions of uninformed shareholders are aligned with the interests of other unbiased shareholders and those of society in general.

The paper has implications regarding measures designed to raise turnout at corporate elections as well. As we have seen above, shareholders’ right to abstain on decisions that they are uninformed about is valuable as long they can leave the decision-making power in the hands of those who better informed. Thus one might expect voter turnout to be low in corporate elections, and in particular, retail shareholders to abstain. However, many corporate decisions require turnout to exceed 50% (or more) in order to be ratified. In

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7Consider for example, the following quotation from the CFO of TELUS corporation, Robert McFarlane, who was faced with a dilemma as a large and potentially partisan voter — Mason Capital — attempted to overturn a vote in favor of consolidating TELUS’s different share classes: “Retail shareholders in particular are notorious for not bothering to vote. Even if you have 90 per cent of non-Mason people vote yes for the proposal it’s possible that the proposal gets defeated because only 60 or 65 per cent of [shareholders] bother to vote.” The company’s response to this problem was to mount a vast campaign to inform small shareholders about the benefits of the proposal, including hiring a proxy solicitation firm to call and e-mail shareholders, and provide information regarding the proposal in the hope that once informed, small shareholders would indeed vote.
order to allow corporations to effect changes in the face of low voter turnout, the US has a rule which allows brokers to vote unvoted shares. While this policy does have the effect of increasing the fraction of votes cast, it reduces welfare by introducing more noise into the outcome which indirectly encourages even more uninformed partisans to vote.

The paper is laid out as follows. In the following section we discuss related literature. In Section 3 we present the baseline model, with its voting equilibrium derived in Section 4. In Section 5 we introduce a model of a vote market and show that trade at zero price can improve welfare compared to the baseline voting equilibrium. In section 6 we extend the model to allow for various frictions including contrarian short sellers and show that (under certain conditions) the presence of frictions shuts down vote market operating at zero price exactly when vote trading is socially detrimental. Section 7 concludes.

2. RELATED LITERATURE

In its aim our paper is most closely related to the small literature on empty voting. This area has been pioneered by Hu and Black (2006, 2007) who explain how voting rights separated from cash flow rights may be acquired using derivative contracts and share borrowing, and uncover real-life examples where hedge funds are known to have voted more shares than they had an economic interest in. These authors also point out that the practice is perfectly legal according to current laws and suggest legal avenues to improve outcomes. Their main policy suggestion is that disclosure requirements should be tightened (i.e., funds should be required to reveal ownership at lower ownership thresholds), and disclosure should be required to occur in a more timely fashion. They do not, however, provide a formal economic theory of empty voting, nor provide evidence that empty voting is an especially common occurrence.

Christoffersen et al. (2007) tackle the latter question by examining two data sets of share lending volumes and prices from the US and the UK. They show that share lending volumes spike around record dates whereas prices do not. They also show that the spike in volumes is more pronounced for votes in which various proxies suggest that there is more asymmetry of information; and that spikes tend to be associated with support for shareholder proposals and opposition to management proposals. They also find that in the UK, dissemination of information through proxy information services appears to be a substitute for large volumes of vote trading. They argue that all of this evidence is suggestive of the idea that uninformed voters deliberately lend their votes to informed voters, and that the fear that such lending might be abused by vote buyers (or borrowers) purchasing votes to vote them against the corporate interest does not seem to be borne out in large data sets. Their empirical paper can be seen as an inspiration to our theory piece which explains how such a vote-trading market can exist in equilibrium.

The existing theory literature on vote trading is sparse. A few early papers on the market for corporate control discuss the optimality of one-share-one-vote. Although they do not
allow for vote trading explicitly, some of their conclusions can be seen as related since vote trading clearly allows departures from the one-share-one-vote rule. Papers in this vein include Harris and Raviv (1988), Grossman and Hart (1988), and Gromb (1992).

Harris and Raviv (1988) ask how votes should be allocated among shares, and whether simple majority or some other rule should be adopted in a context where different management teams may create different amounts of value, but also may extract different amounts of private benefits. They show that one-share-one-vote with majority voting is indeed optimal in their model. The reason is that if some shares have relatively few cash-flow rights, it is relatively cheap for a corporate raider who values control to buy up those votes; but this attracts the “wrong” kind of raider—one who has large private benefits from control, and expects few improvements to cash flow. Grossman and Hart (1988) reach similar conclusions using a different model where shareholders are atomistic and so entirely discount the possibility that their votes sway the election; they show that one-share-one-vote is optimal as long at least one of the candidates has no private benefits of control. Gromb (1992) shows that this conclusion is sensitive to voting shareholders’ assumption that they are atomistic; otherwise there can be a gain to dividing up the votes differently so that there is a class of non-voting (truly atomistic) shareholders and another class of super-voting shareholders who consider their votes important for the election. Blair et al. (1989) argue that vote-buying can enhance the efficiency of contests for corporate control when realized and unrealized capital gains on shares are taxed differently. Burkart and Lee (2008) provide a recent survey of the theoretical literature on this topic; Adams and Ferreira (2008) survey the empirical literature. Gompers et al. (2010) examine empirically whether departures from one-share-one-vote are associated with reduced firm performance. They find that firm performance is increasing in insiders’ (i.e., management’s) cash-flow rights and decreasing in their control rights.

The issue of vote trading is specifically addressed by Neeman and Orosel (2006) and Brav and Mathews (2011). Neeman and Orosel (2006) examine a model where an incumbent CEO (or similar) can behave well or opportunistically, and voters can attempt to discipline him by electing a rival to control the firm if he misbehaves (which is myopically optimal). Both incumbents and challengers may buy votes from voters, and as long as voters are well-informed, it makes no difference in their model whether vote-buying is allowed or not. However, when voters are not well-informed, candidates can use their willingness to pay for votes as a signal. They show that if more talented candidates for CEO are willing to pay a higher price for votes then vote-trading can enhance efficiency. This model thus applies particularly to the case of elections of corporate personnel, and is obviously very different to the one presented here which applies more generally.

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\textsuperscript{8}Zachariadis and Olaru (2013) examine the related issue of voters’ mixed incentives in corporate restructurings when they may hold both debt and equity in a firm.
Like us, Brav and Mathews (2011) are interested in the issue of whether the practice of “empty voting” is beneficial in corporate contexts more generally. Their model, however, is very different. It examines the trading behavior of an informed, strategic agent (a “hedge fund”) with initially a zero stake in the decision. It is assumed that ownership of shares acquired before the record date is publicly disclosed (and so generates no trading profits), whereas shares acquired secretly after the record date cannot be voted. The hedge fund can reap trading profits by delaying the acquisition of shares until after the record date (when it has acquired information) and vote borrowed or purchased votes instead. The key decision is therefore whether to buy shares or simply votes before the record date. In contrast, we study vote trading while taking voters’ initial positions (preferences or bias) as well as their heterogenous information about the state as given. In the model of Brav and Matthews (2011) the supply of votes (stripped from shares) is exogenous, and so is the voting behavior of shareholders other than the hedge fund of interest. In contrast, we fully endogenize the supply of shares and the voting behavior of other shareholders. Brav and Matthews (2011) do not directly address the question of whether the ability to acquire votes without shares raises or decreases social welfare: instead, they ask whether the presence of the hedge fund is advantageous for shareholders. They find that since the hedge fund needs to hide its trades in order to profit from them, it may, on occasion, deliberately sell short after the record date and use its borrowed or purchased empty votes to reduce shareholder value. The hedge fund’s presence is more likely to lead to a welfare improvement if it is more costly to buy votes without shares, because then this value-destroying strategy becomes more costly. Shareholders are also more likely to benefit if they are less well-informed about the state (i.e., their random votes are less correlated with the state). In contrast, we find that the value-destroying effect of vote trading may be mitigated by endogenous vote supply.

Our paper is also related to the economics and political science literature on voting incentives and outcomes. In seminal contributions, Feddersen and Pesendorfer (1996) and Austen-Smith and Banks (1996) showed that uninformed, unbiased voters may strictly prefer to abstain in order to cede decision-making to those who are informed and unbiased. A natural question to ask is whether social outcomes could be further improved by allowing the trading of votes. We provide the answer in a framework similar to that of Feddersen and Pesendorfer (1999), with a random number of voters (Poisson distributed with a finite mean) and heterogeneous preferences. A centralized market for votes may aggregate information just as (or even better than) communication; in this sense our paper is related to models of deliberation, e.g., Gerardi and Yariv (2007).

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9This is realistic, for example, in situations where the float of shares to purchase is more limited than the supply of shares available for borrowing (e.g., if a large chunk a company is owned by institutions that buy and hold for the long term but lend their shares out). Christoffersen et al (2007) find no evidence of vote-trading going on in the share market itself.

10Other recent papers that adopt a similar Poisson framework include Krishna and Morgan (2011, 2012), Bouton and Castanheira (2012), and Bouton (2013).
We are not aware of any papers in the literature that introduce vote trading with both preference and information heterogeneity. Casella et al. (2012) consider a market where voters differ only in terms of preferences, i.e. a pure private values setting. The equilibrium of their model resembles a second price auction for control rights: the voter with the highest valuation demands a majority of votes at a price that makes the second-highest valuation voter indifferent between supplying and demanding. In general, such an outcome reduces utilitarian social welfare compared to no trading. The supply motive we identify is not present in their model since there is no information heterogeneity.

Goeree and Zhang (2012) and Weyl (2013) also consider the private values setting, and introduce a mechanism designer who can create new votes to sell. Both show that versions of an expected externality mechanism can improve welfare relative to simple voting. In our paper, trade is also facilitated by a market maker (quoting buy and sell prices), but we assume that all supply must be endogenously generated by the trading mechanism itself — votes that do not already exist cannot be created.

3. Baseline Model

We begin by describing a voting model in a corporate setting into which we later introduce the possibility of exchanging (buying and selling) votes. It is important to emphasize that the presentation of the model in terms of shareholders and a firm is merely for concreteness. The theoretical framework we adopt is common in many areas of political economy, most notably the analysis of political elections with two candidates.11

The decision to be made is \( d \in \{a, b\} \), e.g., whether or not to merge with a target firm. Each shareholder has a single share and the corresponding vote, which he or she can either cast for \( a \), cast for \( b \), or discard (abstain). Payoffs from the outcome of the vote are affected by the state of nature, \( \omega \in \{\alpha, \beta\} \), with \( \Pr[\omega = \alpha] = \Pr[\omega = \beta] = \frac{1}{2} \). The probabilities, but not the realization of \( \omega \), are commonly known; the two states are equiprobable for simplicity. The outcome is decided by simple majority voting with the rule that in case of a tie the decision not matching the state prevails.12

The common-value component of the shareholders’ payoff is normalized to be 1 per share owned from the decision matching the state and \((-1)\) from the decision not matching the state. The interpretation is that decision \( a \) is “right” in state \( \alpha \) and \( b \) in state \( \beta \). For example,

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11The model is closest to that of Feddersen and Pesendorfer (1999). The Poisson voting model is combined with a simpler (common-values) preference structure in, e.g., Krishna and Morgan (2012). Preference heterogeneity is maintained with a continuum of (behaviorally biased) voters in Levy and Razin (2014). Many other applications are cited in these recent papers.

12Other tie-breaking rules used in the literature include 50-50% randomization or assigning of the decisions as the “default”. Our proposed rule (in a tie, the “wrong” decision prevails), or its opposite, greatly streamlines the presentation without affecting the intuition and the results. It could be “micro-founded” by assuming that half a vote is cast against the state (by contrarians) — the point of this (or a “default”) assumption is essentially to assume away ties.
if $a$ corresponds to a takeover then the takeover creates value (for the shareholders voting on the decision) in state $\alpha$ and destroys value in state $\beta$.

In addition, each shareholder enjoys a randomly-drawn private benefit (gain or loss) $\tau \in \mathbb{R}$ from the majority decision being $d = a$, and benefit $-\tau$ from $d = b$. We refer to $\tau$ as the shareholder’s private preference type. This heterogeneity may be interpreted as each shareholder’s level of exposure to a monetary transfer conditional on decision $a$ (e.g., the merger) irrespective of whether or not it is value-enhancing; for ease of exposition this transfer is reversed when $d = b$. Alternatively, $\tau$ may be thought of as a “bias” that the shareholder has in favor of a particular outcome (here, decision $a$). For example, shareholders who are also managers or employees in the firm might have innate biases for or against a merger that are not shared by other shareholders. We call a shareholder with $\tau > 0$ an $a$-partisan and one with $\tau < 0$ a $b$-partisan; $|\tau| \leq 1$ is a weak partisan wanting to induce $d$ to match $\omega$ irrespective of the private gain.

Our baseline model will feature two kinds of strategic shareholders: ones that are informed of the state and ones that are uninformed; they are all assumed to be weak partisans. We will also introduce (a random number of) noise voters who cast their votes for $a$ and $b$ at random, irrespective of the state. This is both for the sake of realism, and also to represent strong partisans voting for their preferred decision even if it is against the common good.

Specifically, we assume there is a fixed number, $K \geq 1$, of shares and corresponding votes in the hands of strategic weak partisan shareholders informed of $\omega$. (It is commonly known that they all know the state.) Neither their preference parameters, $\tau \in [-1, 1]$, nor the concentration of their $K$ shares/votes in few of many informed hands will play any role in the analysis, therefore we keep these aspects of the model unspecified.\footnote{In the baseline voting equilibrium all $K$ informed votes will be cast for the decision that matches the state, no matter what the $\tau$’s and the ‘market power’ of informed weak partisans. When a market for empty votes is introduced, the valuations of informed shareholders for empty votes are affected by these details, but not the equilibrium price of a vote in the specific mechanism that we consider below.}

In addition, there are $M$ strategic weak partisan shareholders (each with one share, one vote) that are uninformed of the state; they will be at the center of our equilibrium analysis. Each such voter’s preference type is drawn i.i.d. uniform on $[-T, T]$, i.e., for $y \geq 0$,

$$\Pr[-y \leq \tau \leq y] = \pi(y) = \begin{cases} 
y/T & \text{if } y \leq T \\
1 & \text{otherwise,} \end{cases}$$

where $T \leq 1$ is a commonly known parameter. We assume that the number of uninformed weak partisans, $M$, is itself a Poisson random variable with mean $\mu$. The assumption that $M$ is random reflects the real-world phenomenon that not all well-meaning shareholders uncertain of the benefits of a looming merger learn or realize on time that there will be a vote (e.g., some may fail to obtain a ballot, etc.). The Poisson assumption, which is
commonly used in the voting literature, is mostly innocuous and makes this exogenous randomness tractable in the model.\footnote{In particular, due to the Poisson assumption, an uninformed weak partisan that is present in the model perceives the model exactly as we (modelers, outside the model) do. This \textit{environmental equivalence} property makes the equilibrium and welfare analysis a lot more expedient.}

Finally, we assume there also exist $N$ noise voters, each of whom will vote for $a$ or $b$, independently, with 50-50% chance. Let $N$ be Poisson with mean $\nu$, drawn independently of $M$. As mentioned above, besides idiosyncratic noise, these voters could also be thought of as \textit{strong partisans} (shareholders with $\tau > 1$ or $\tau < -1$), present in a random number, and excluded from welfare calculations.

There are three interesting and realistic extensions that we will discuss throughout, even though, for the benefit of clarity and brevity of exposition, these are not specifically included in the baseline model. First, we will allow that (e.g., with exogenous probabilities reflecting frictions) cast votes may be lost and/or abstaining votes may be cast (e.g., \textit{forced voting}).\footnote{In the U.S., if individual shareholders do not vote their shares then their brokers may have the right to do so on their behalf. This is referred to as “broker vote”, which is a form of forced voting.} Second, we will consider informed or uninformed \textit{strong partisans} (shareholders that vote for a fixed outcome irrespective of the state).\footnote{As noted above, noise voters could be thought of as strong partisans whose welfare is ignored.} Third, we will analyze the effects of \textit{contrarians}: shareholders that are informed of the state and consistently vote for the opposite. Note that the latter are conceptually different from strong partisans as their preference depends on the state. We note that certain symmetry assumptions (e.g., concerning the distribution of $\tau$) could be relaxed at the expense of additional notation.

\section*{4. Voting Equilibrium}

In this section we derive the equilibrium of the baseline voting model (without vote trading), and discuss how the equilibrium changes as a function of the model’s parameters.

\subsection*{4.1. Threshold equilibrium and aggregate outcome}

Recall that there are $K \geq 1$ shares (hence votes) in the hands of weak partisans informed of the state; $N \sim \text{Poisson}(\nu)$ in the hands of noise voters, and $M \sim \text{Poisson}(\mu)$ in the hands of uninformed weak partisans.

Given our assumptions, irrespective of the preference parameters of informed weak partisans, it is a dominant strategy for them to vote for the decision that matches the state ($a$ in state $\alpha$, $b$ in state $\beta$). Noise voters by definition vote for $a$ or $b$ with 50-50% chance.

The only strategic decision left to be considered is that of an uninformed weak partisan. It is easy to see that any such actor with preference parameter $\tau$ will use a threshold strategy, voting for $a$ iff $\tau > \tau^*$, voting for $b$ iff $\tau < -\tau^*$, and abstaining iff $\tau \in [-\tau^*, \tau^*]$.\footnote{Here and throughout we use the convention that an indifferent voter abstains.}

This is so because the private benefit from voting for $a$ (preference type $\tau$), is additive in the shareholder’s payoff, hence (for instance) if a shareholder with type $\tau$ prefers to
vote for $a$ then another with $\tau' > \tau$ prefers to vote for $a$ as well, provided they share the same beliefs about the state and the others’ voting behavior. However, in equilibrium, all uninformed shareholders share the same beliefs about all relevant events.

As a result, in equilibrium, the number of votes for outcome $a$ in state $\alpha$ (equivalently, for $b$ in state $\beta$) is $K + X$, such that $X$ is a Poisson random variable with mean $x/2$, where

$$x = \nu + [1 - \pi(\tau^*)] \mu$$

is the mean value of the number of votes cast at random (for $a$ and $b$ with 50-50% chance), computed as the sum of the expected number of noise voters, $\nu$, and the expected number of uninformed weak partisans whose $\tau$ lies outside the $[-\tau^*, \tau^*]$ interval, i.e., $[1 - \pi(\tau^*)] \mu$. The number of votes for $b$ in state $\alpha$ (equivalently, for $a$ in state $\beta$) is simply another (independent) Poisson random variable with mean $x/2$.

Even without deriving the equilibrium threshold $\tau^*$ we are able to consider the effect of various parameters on the probability the decision matches the state. Clearly, when $K$ increases, this probability increases, all else equal. The other parameters affect this probability via changing the mean $x$ of the Poisson distribution defined above. Intuitively, when $x$ increases the noise in the election also increases, which should decrease the probability of a correct decision. The next result formalizes this intuition.

**Lemma 1.** The probability that the decision matches the state is strictly decreasing in $x$.

The formal proof is in the Appendix. We use this result below to assess welfare.

4.2. **Properties of the baseline voting equilibrium.** An uninformed weak partisan’s decision (whether to vote $a$, vote $b$, or abstain) makes a difference for his or her payoff when such decision is *pivotal*. The events when a vote is pivotal are when there is a *tie* (in which case a vote for the state changes the decision to match the state), and when the decision matching *the state is one vote ahead* (in which case a vote against the state induces the decision not to match the state). Next, we calculate the respective probabilities of each of these events, and then characterize the shareholder’s optimal choice.

The probability that the realization of a Poisson random variable with mean $x/2$ is exactly $n$ (where $n = 0, 1, 2, \ldots$ is a fixed, natural number) is $e^{-x/2} (x/2)^n / n!$. Therefore the probability that in a given state of nature the number of votes cast for the decision matching the state is exactly $n + K$ is $e^{-x/2} (x/2)^n / n!$ whereas the probability that there are exactly $n + K$ votes against the state is $e^{-x/2} (x/2)^{n+K} / (n + K)!$. (The difference in the two expressions is due to the fact that $K$ sure votes are cast for the state in addition to the random ones.) Since the random votes for and against the state are independent, the probability that in a given state there is a tie (at any number of votes) is

$$P_0 = e^{-x} \sum_{n \geq 0} \frac{(x/2)^n (x/2)^{n+K}}{n! (n + K)!} = e^{-x} I_K(x),$$
where \( I_K(x) \) is the \( K \)th-order modified Bessel function of the first kind. This function is widely used and hence profoundly understood in the applied mathematics and physics literature (see, e.g., Olver, 1970), and its useful properties, discussed below, will be relied on in the analysis of our model as well.

Similarly, the probability that the decision matching the state is “one vote ahead” in a given state (either state) is

\[
P_+ = e^{-x} \sum_{n \geq 0} \frac{(x/2)^n(x/2)^{n+K-1}}{n!(n+K-1)!} \equiv e^{-x}I_{K-1}(x),
\]

Given a (conjectured) equilibrium abstention threshold for uninformed weak partisans, \( \tau^* \), a voter with \( \tau \geq 0 \) prefers abstaining to voting for \( a \) if, and only if,

\[
\frac{1}{2} \left( P_0 \right) \sum_{\omega = \alpha \text{ vote pivotal}} 2(1 + \tau) + \frac{1}{2} \left( P_+ \right) \sum_{\omega = \beta \text{ vote pivotal}} 2(\tau - 1) \leq 0.
\]

If the state is \( \alpha \) (which has 50% chance according to an uninformed voter), casting a vote for \( a \) rather than abstaining pivotal for decision \( a \) whenever a vote for \( a \) breaks a tie; in this case the shareholder’s gain is \( 2(1 + \tau) \). In contrast, if the state is \( \beta \), then voting for \( a \) is pivotal for \( a \) if and only if the state would have been one vote ahead; then the voter’s gain (in fact a loss) is \( 2(\tau - 1) \). If the expected net gain is non-positive then the voter chooses abstention.

Using the definitions of \( P_0 \), and \( P_+ \) and letting \( x(\tau^*) = \nu + |1 - \pi(\tau^*)|\mu \), the above condition for an uninformed shareholder to abstain can be rewritten as

\[
(1 + \tau)I_K(x(\tau^*)) - (1 - \tau)I_{K-1}(x(\tau^*)) \leq 0. \tag{4.1}
\]

An abstention threshold \( \tau^* \) is part of a voting equilibrium provided (4.1) holds exactly for \( \tau \in [0, \tau^*] \). Our first result establishes the existence and uniqueness of a voting equilibrium with non-trivial abstention and derives the comparative statics of \( \tau^* \) in the model’s parameters.

**Proposition 1** (Voting equilibrium with abstention). For any \( \nu, \mu > 0 \), \( K \geq 1 \) and \( T \in [0,1] \), there exists a unique voting equilibrium with \( \tau^* \in (0,1) \). Uninformed weak partisans vote for \( a \) iff \( \tau > \tau^* \), for \( b \) iff \( \tau < -\tau^* \), and abstain otherwise.

The equilibrium abstention threshold \( \tau^* \) is increasing in \( K \) (more abstention for more informed voters), decreasing in \( \nu \) and \( \mu \) (less abstention for more noise), and decreasing in \( T \) (less abstention for a more spread-out distribution of \( \tau \)).

The proof is relegated to the Appendix. The argument for the existence of equilibrium with some uninformed weak partisans abstaining is based on the observation that if no such shareholder abstained then an uninformed voter with \( \tau = 0 \) would strictly prefer to abstain, fearing that his or her vote will be more likely to be pivotal against a decision
matching the state than for it. This is the well-known effect of the *swing voter’s curse* identified by Feddersen and Pesendorfer (1996). Formally, when $\tau^* = 0$, the left-hand-side of inequality (4.1) is strictly negative, hence $\tau^*$ must rise. At the other extreme, an uninformed voter with $\tau = 1$ strictly prefers to vote for $a$ (rather than abstain), because even in state $\beta$ the loss from inducing the wrong decision is nil for this voter (formally, inequality 4.1 strictly fails at $\tau = 1$). By continuity, there must exist an equilibrium threshold $\tau^* \in (0, 1)$. Uniqueness of the equilibrium as well as the comparative statics follow from the properties of Bessel functions. 18

As for a numerical example: if $K = 1$ (a single informed vote), $\nu = \mu = 1$ (in expectation one noise voter and one uninformed weak partisan voter), $T = 1$ ($\tau$ uniform on $[-1, 1]$), the unique voting equilibrium threshold is $\tau^* \approx 0.2$.

### 4.3. Interpretation and consequences of the results.

It should be noted that from the perspective of weak partisans, or from that of an uninformed and unbiased social planner, before the realization of preference types, the socially optimal voting outcome would be *abstention by all uninformed shareholders*, i.e., $\tau^* = 1$. This is so because none of these shareholders knows the state, hence their votes do not help to aggregate information about the source of the common-value component of their welfare; in addition, their private biases offset in expectation. Hence, by Lemma 1, the voting equilibrium that has $\tau^* < 1$ is *inefficient*: it does not aggregate all available information because some uninformed voters add noise to the outcome. On the bright side, however, some uninformed voters always abstain ($\tau^* > 0$).

The comparative statics of the equilibrium are quite intuitive. A larger number of informed votes $K$ (cast for the decision matching the state) exacerbates the swing voter’s curse, hence an uninformed voter feels compelled to abstain even with a stronger bias: $\tau^*$ increases. The opposite effect prevails when the number of “random votes” increases, due to more noise voters (higher $\nu$) or more uninformed weak partisans that vote (either higher $\mu$ or a more spread-out distribution of $\tau$). The derivation of these results relies on properties of the Bessel functions, which are summarized in Lemma A.1 of the Appendix.

Finally, let us consider the extensions mentioned at the end of the modeling section. First, if abstention resulted in the vote being cast at random with some probability (as in the case of “forced voting” or “broker vote”), then this would reduce the benefit of abstaining and, all else equal, reduce the equilibrium level of $\tau^*$. Second, the presence of strong partisans, which could be modeled as an increase in noise parameter $\nu$, would also result in a lower equilibrium $\tau^*$. Third, contrarians (simply modeled as a reduction in $K$) would also lower the equilibrium level of $\tau^*$. All of these changes would reduce the ex-ante expected welfare in the voting equilibrium.

---

18 The model is most closely related to the Poisson voting model of Feddersen and Pesendorfer (1999). Our slightly more specific assumptions (e.g., binary state and signals) allow us to establish the uniqueness of the abstention equilibrium threshold, which we require in order to obtain global comparative statics results.
5. A Market For Empty Votes

In this section we show that allowing a market for votes prior to voting improves information aggregation and thereby efficiency. The key reason for this result is that informed shareholders have a positive willingness to pay for additional votes in such a market, whereas uninformed weak partisans (even those who would vote sans trading) have strictly negative valuations, provided they believe that supplied votes will end up in the hands of informed shareholders. An appropriate (in fact, zero) price can separate these two groups into buyers and sellers.

An informed shareholder’s willingness to pay (per share owned), with preference type \( \tau = 0 \), for the marginal vote when \( K + D \) votes are anticipated to be cast for the state by informed shareholders and \( \tilde{X} \) at random by uninformed shareholders or noise voters is

\[
r(D) = 2 \Pr[\tilde{X} = K + D] > 0.
\]

This is (two times) the probability that the marginal vote bought by the informed shareholder becomes pivotal and flips the decision to match the state. For an informed shareholder with \( \tau \neq 0 \) the valuation of the marginal vote per share already owned is \((1 + \tau)r(D)\) in state \( \alpha \) and \((1 - \tau)r(D)\) in state \( \beta \), which remains positive for any \( \tau \in (-1, 1) \).

Note that in a fully efficient scenario all votes held by uninformed shareholders are bought by someone informed (and consequently only noise voters cast votes at random), therefore \( D = M \) and \( \tilde{X} \) follows a Poisson distribution with mean \( \nu \).

Of course, it does not follow that an informed shareholder will necessarily make a bid proportional to his or her willingness to pay (times the number of shares owned) for every available vote. Buying up the decision rights of uninformed shareholders is a pure public good, so informed voters may try to free-ride on each other’s purchases. This will drive the price down — though how far down depends on the exact trading mechanism.

An uninformed weak partisan would always strictly prefer his or her vote(s) to be cast by someone else who is informed for the state; so if he or she can trust that all votes are bought by (and only by) informed shareholders, then his or her valuation for a vote (own or acquired) is non-positive. Indeed, fearing that his or her demand may crowd out that of an informed shareholder, an uninformed weak partisan would not even want to acquire an empty vote (supplied by an uninformed actor) if it was free.

Next, we describe our model of a market for empty votes as a specific, two-stage game with voluntary vote trading preceding the actual vote. In order to add to the richness of the model, parameter \( \rho \in [0, 1] \) is introduced measure the exogenous, institutional “reach” of the trading mechanism: We assume that a proportion \( \rho \) of the uninformed weak partisans are made aware of the opportunity of buying and selling empty votes prior to voting. (An alternative interpretation is that \((1 - \rho)\) proportion of supplied votes are exogenously returned to their owners.) Importantly, uninformed shareholders told of the trading opportunity are not required to supply or demand votes. All informed shareholders are
aware of the option to buy or sell votes prior to voting; noise voters are excluded from vote trade. The trading outcome is publicly disclosed, after which all shareholders with retained voting rights are invited to vote.

In the vote-trading stage each market participant (comprising a $\rho$ proportion of $M$ uninformed weak partisans and all $K$ informed shareholders) submits either a “bid” or an “ask” schedule: a valuation per vote as a function of the net number of votes demanded or supplied by him or her. Formally, for shareholder $i$, this is denoted by $r_i(d_i)$, where $d_i \in \mathbb{Z}$ is the net number of votes that $i$ receives ($d_i < 0$ represents votes supplied), and $r_i(d_i)$ is $i$’s reported valuation for each vote (sold or acquired depending on the sign of $d_i$). An equilibrium price $r^* \geq 0$ either nets zero aggregate excess demand, $\sum_i r_i^{-1}(r^*) = 0$; or in case of imbalance at $r^* = 0$ the short side of the market is evenly rationed. After the market clears, all votes bought or retained are cast in a majority vote.

Shareholders are assumed to act strategically both in the vote trading phase and the subsequent vote. They update their beliefs about other voters’ information and future behavior rationally and act accordingly; formally, we derive a Perfect Bayesian Equilibrium of the two-stage game.

The following proposition establishes the existence of an equilibrium in which all uninformed weak partisans that are initially aware of the vote market supply their vote, and all supplied votes are bought by informed shareholder(s). A striking feature of the equilibrium is that the price of a vote is zero. Uninformed weak partisans who are not invited to trade abstain from the subsequent vote with a wider range of preference bias parameters than they would have had the vote market not existed. If all uninformed voters are aware of the market for empty votes ($\rho = 1$) then the equilibrium outcome is fully efficient.

**Proposition 2** (Equilibrium in the vote market). The vote-trading model has an equilibrium such that all uninformed weak partisans invited to the market supply their vote at zero price, and all informed shareholders bid zero for any number of votes available on the market.

As a result a fraction $\rho$ of the votes held by uninformed shareholders are acquired and voted for the state by informed shareholders. A uninformed weak partisan who is either not invited to trade or one that retains his or her vote out of equilibrium abstains from voting iff $|\tau| \leq \hat{\tau}$, where $\hat{\tau} > \tau^*$.

---

19 A shareholder is not allowed to sell more votes than the number of shares owned; that is, $d_i$ must weakly exceed the negative of the number of $i$’s shares.

20 An alternative solution concept would be a Walrasian equilibrium in the market for empty votes, followed by a Bayesian (game-theoretic) equilibrium at the second, voting stage. However, this might be considered conceptually disconnected: in a market equilibrium agents are assumed to ignore exactly the effect (that they might be pivotal, non-atomic) which they condition on in a strategic voting equilibrium.

21 This is not at all at odds with reality. Christofferson et al. (2007) have found that, even though share lending activity peaks around the record date, the cost of borrowing shares remains flat. The cost of borrowing is positive due to factors omitted from our model, e.g., the requirement of short sellers to borrow shares before their short position is covered. The cost (in terms of percentage return) is about 1% per year, which is negligible for the duration that a share needs to be borrowed before a vote.
Social welfare is improved (due to information being better aggregated) by vote trading followed by voting than by voting alone. The exact way this works is the following. Uninformed weak partisan shareholders suffering from the swing voter’s curse strictly prefer to give up their own vote to an informed shareholder who will cast it for the state. Even those with preference parameters more extreme than \( \tilde{\tau} \), who continue to vote the shares that they hold, are not buying additional votes (even at zero price) because they do not want to “crowd out” the demand of informed shareholders. Informed shareholders are willing to pay a positive price for empty votes; however, since purchasing and casting empty votes for the state is a pure public good, “free riding” drives the price to zero.\(^{22}\) Finally, an indirect and subtle source of efficiency gain is that uninformed weak partisans abstain for a wider range of preference parameters than they would in the absence of a vote-trading market (for \(|\tau| \leq \tilde{\tau}\) such that \(\tilde{\tau} > \tau^*\)). This is so because they are aware that vote trading reduces the “noise” in the voting outcome (it is more likely that informed shareholders prevail), which exacerbates their swing voter’s curse.

The efficiency of the vote-trading market (subject to all uninformed shareholders being present in the market), especially at a zero equilibrium price for empty votes, should be thought of as a benchmark result. The key condition is that uninformed weak partisan vote suppliers know that their empty vote, once sold, is not going to be “diverted” to a shareholder who will cast it at random (based on their own personal preference), or even worse, against the state. Thus, the first insight that emerges from our formal analysis is that in the absence of such frictions it is indeed socially beneficial to allow vote trading. In the next section we introduce frictions (exogenous, technical dissonances as well as conflicted vote buyers who act against the unbiased shareholder’s interest), and investigate whether these can reverse our conclusions.

6. Frictions and Conflicted Buyers

In this section we depart from the benchmark vote-trading model of the previous section by introducing certain types of frictions and conflicted buyers. These will be modeled as exogenous probabilities of supplied votes being destroyed (lost), returned to the supplier, diverted to be voted for a decision at random (from the uninformed vote seller’s perspective), and diverted to be voted by a contrarian specifically against the state.

In what follows the population of shareholders in the voting game (and the preceding vote market) remains the same as before, with \(K \geq 1\) informed and \(M\) uninformed shareholders, as well as \(N\) noise voters. We simplify the model by setting all informed and

\(^{22}\)Not all trading mechanisms would lead to exactly a zero price. For example, suppose a market maker can buy and sell votes at pre-committed “bid” and “ask” prices. The fact that uninformed shareholders have negative valuations for votes in the equilibrium described in Proposition 2 whereas the informed have positive valuations implies that the market maker can pick positive prices with a positive spread that induce the same outcome (all votes of uninformed shareholders transferred to informed ones) while making money on the traded shares.
uninformed shareholders’ preference types equal to zero ($\tau \equiv 0$), and also for simplicity we set the reach (or prevalence) of the vote-trading market to $\rho = 1$.

Specifically, we assume that in the vote-trading phase a vote supplied may be “hijacked” as follows: with probability $\lambda_0$ it is destroyed or returned to the supplier; with probability $\lambda_1$ the supplied empty vote ends up in the hands of a strong partisan who votes it at random for $a$ or $b$; whereas with probability $\lambda_2$ the empty vote is acquired by a contrarian and cast against the state. With probability $(1 - \lambda_0 - \lambda_1 - \lambda_2)$ the vote ends up in the hands of an informed non-contrarian shareholder and it is voted for the state. We also allow a fixed number of shares, $L \geq 0$, to begin the game in the hands of contrarians.

The main question we want to answer in this section is whether in the presence of these frictions opening up the vote market can reduce welfare, especially, by allowing conflicted agents (contrarians) to gain votes.

The somewhat surprising answer to this question is that the vote-trading market is, in a sense, self-regulating: An equilibrium in which all uninformed shareholders tender their votes at zero price (just like in the case without frictions) continues to exist if, and only if, this improves welfare compared to the case with no trade. Otherwise, if vote trading at zero price would be detrimental to social welfare, then uninformed and unbiased shareholders will simply not supply their vote at zero price, and the market (with zero supply price) features no trade.

In order to see why this is the case, suppose that in equilibrium all uninformed shareholders supply their vote at zero price, to be acquired by informed shareholders (unless diverted by frictions). Then the number of votes cast for the state is $K$, plus a Poisson variable with mean $x_K = \frac{1}{2} \nu + \frac{1}{2} \lambda_1 \mu + (1 - \lambda_0 - \lambda_1 - \lambda_2) \mu$.

In contrast, the number of votes cast against the state is $L$ (the number of votes controlled by contrarians), plus a Poisson variable with mean $x_L = \frac{1}{2} \nu + \frac{1}{2} \lambda_1 \mu + \lambda_2 \mu$.

If a given uninformed voter deviates from the hypothesized equilibrium (does not supply at zero price), then the probabilities of a tie ($\hat{P}_0$) and the state being one vote ahead ($\hat{P}_+$) are determined by the distributions of these random variables. For example,

$$\hat{P}_0 = e^{-x_K-x_L} \sum_{n \geq 0} \frac{x_K^n x_L^{n+K-L}}{n!(n+K-L)!} = e^{-x_K-x_L} \left( \frac{x_L}{x_K} \right)^{K-L} I_{K-L}(\sqrt{x_K x_L}).$$

$\hat{P}_+$ can be computed analogously.

If the shareholder does supply his or her vote, then the number of votes for the state increases by one with probability $(1 - \lambda_0 - \frac{1}{2} \lambda_1 - \lambda_2)$, and the number of votes against the state increases by one with probability $(\frac{1}{2} \lambda_1 + \lambda_2)$.

---

23 Since uninformed shareholders now all have preference type $\tau = 0$ they will abstain a returned vote. As a result the effect of returning or destroying a supplied vote is the same.

24 Contrarians are different from strong partisans: they know the state and prefer to vote against it.
Therefore, an unbiased uninformed shareholder weakly prefers to supply his or her vote at zero price (as hypothesized), whenever

\[
(1 - \lambda_0 - \frac{1}{2}\lambda_1 - \lambda_2) \hat{P}_0 - (\frac{1}{2}\lambda_1 + \lambda_2) \hat{P}_+ \geq 0. \tag{6.1}
\]

Note that by the assumption of an uninformed shareholder being unbiased (having preference type \( \tau = 0 \)), the condition for this shareholder wanting to supply his or her vote at zero price is equivalent to this action being socially beneficial; both are expressed by inequality (6.1).

The following proposition summarizes the conclusions reached thus far: In the presence of frictions the vote-trading market has an efficient equilibrium provided (6.1) holds. In contrast, if (6.1) fails then there is no equilibrium in which a positive amount of empty votes are traded at zero price.

**Proposition 3 (Equilibrium with frictions).** In the model with frictions and \( \tau \equiv 0 \), if condition (6.1) holds, then the vote-trading market has an equilibrium in which uninformed shareholders supply and informed ones demand empty votes at zero price. Votes are supplied at zero price if, and only if, doing so increases welfare.

This result — that trading votes at a zero price can only increase and never decrease welfare — is striking. It can be considered as a useful benchmark against which to measure the concerns of regulators and lawyers. However, it does rely on several assumptions. In the remainder of this section, we discuss the limitations and implications of the result.

First, we have assumed that vote-supply is done knowingly and voluntarily. In some realistic circumstances, this may not be the case. For example, as mentioned in Section 4 above, under some circumstances brokers are able to vote their clients’ shares when these have not been voted (the so-called “broker vote”). Moreover, when brokers’ clients have purchased shares on margin, the small print to this contract generally allocates the right to lend out those shares to the broker. Thus shares bought on margin (and hence the votes attached to those shares) may be loaned out over the record date without the beneficial owner’s knowledge. Our analysis suggests that greater transparency in such practices would be useful in order that shares are always loaned intentionally, rather than unknowingly. Then, with a zero price for share loans made over the record date, the vote-supply market could be “self-regulating”, with votes supplied at zero price only when this is welfare-enhancing.

Second, we have assumed in this section that uninformed voters are unbiased. If these voters were instead weak partisans as in section 5, then this bias would make them less willing to supply than the unbiased voter considered in equation (6.1), because they would attach more value to retaining and using their vote. Therefore, extending the model in this way would, in a sense, strengthen the result that opening a market for votes at a zero price can never reduce welfare, because weak partisans are less willing
than the social planner to supply votes (suggesting that there might be too little trade on such markets, but never too much).\textsuperscript{25}

Third, one may ask what might happen in the voting market with a non-zero price. Empirically, this discussion may be moot for the large majority of cases since Christoffersen et al. (2007) show that the mean price of acquiring a vote through share-lending is zero. This is fortunate since any payment the uninformed supplier of a vote considered in equation (6.1) receives in exchange for supplying drives a wedge between his private value of supplying and the social value of doing so. The problematic effects of vote-trading with private rather than common values has been documented by Casella et al. (2012), so we do not pursue them here. The contribution of our paper is rather to point out the social value of vote-trading in a different setting where it is useful to aggregate information about the best decision and players share common values. By extension, in an environment with a mixture of private and common values, the welfare effect of vote-trading is likely to be in general ambiguous.\textsuperscript{26}

In our model, the welfare impact of vote-trading at a non-zero price depends on the likely purchaser of such votes (informed, contrarians or random strong partisan voters). This turns not only on the probabilities of each type but also on their relative willingness-to-pay. These depend in familiar ways on the distribution of starting votes, and on the concentration or dispersion of votes among shareholders of each type (which determines the extent of free-riding on each side). In this paper, we laid the foundations for such an analysis, but do not undertake it because ultimately, given any set of assumptions about these matters, what we have shown in this section is that trading “empty” votes can reduce welfare only if the enfranchised electorate allow it to do so by supplying their votes; and at a zero price, there is an incentive to supply only supply increases welfare.\textsuperscript{27} Thus a policy prescription of our model is that one way to avoid any harmful effects of vote-trading would be to enforce a zero supply price in the vote-market.

\textsuperscript{25}A difficulty arises, however, if these voters are sufficiently biased that they might want to purchase votes on such markets. It can be shown, however, that when weak partisans preference intensity is the same as that of the informed voters, their willingness to pay is less than that of informed voters because of the “swing bidder’s curse”: weak partisans do not want to outbid an informed voter who will vote for the state. Thus as long as informed bidders are sufficiently active demanders in the vote market, weak partisans would not want to participate on the demand-side.

\textsuperscript{26}Brav and Matthews (2011) study a model of vote trading in which an informed hedge fund may acquire shares on the record trade and then trade afterwards to either a net long or a net short position; in this case, separating votes from shares may either improve or reduce firm value in expectation depending on parameters. This model differs from ours in that it features a single monolithic vote-purchaser and an entirely exogenous supply of votes.

\textsuperscript{27}At a positive price, small shareholders might be tempted to free-ride on the social good by supplying their votes even if this reduces welfare; whereas large shareholders are more likely to withdraw their shares when this is desirable. Such behavior is consistent with increasing adoption of policies to withdraw shares from the share-lending market around record dates (see Aggarwal, et al 2012 for a detailed discussion).
7. Conclusions

In this paper, we have demonstrated a mechanism by which vote trading improves welfare despite the presence of biased voters. Allowing vote trading at a zero price (as is common in practice, Christoffersen et al, 2007) enables informed voters pick up votes in the market, increasing their power in the election. This directly improves welfare by increasing the likelihood that the voting decision is correct (i.e., matches the state) rather than being determined by noise or by voters unconcerned about the correct decision (so-called strong partisans). Meanwhile, welfare also improves indirectly because biased voters who are ignorant of but nevertheless care about making the correct decision (so-called weak partisans) are discouraged from voting in favor of their bias, because they know that they are more likely to be voting against informed voters than in the absence of vote-trading.

In deriving this result, it is important, however, that compensation to sellers is low. This suggests that it may be desirable to absorb any rents from vote-trading through taxation or intermediaries. If the compensation to vote sellers were to rise too high, then sellers might be tempted to provide their vote in circumstances in which vote trading would be damaging, imposing negative externalities on other voters. In Casella et al (2012), for example, there is no information reason to trade votes as voters all know the best decision for them (so there would be no trade at a zero price); but in this case, a voter can pay above the odds for a majority of votes in order to take an action that reduces welfare for the minority of voters whose votes are not purchased, potentially reducing utilitarian welfare.

Our paper is the first to allow for an endogenous vote supply by uninformed but potentially biased shareholders to potentially informed and/or biased voters. Its message is that the presence of biased voters does not negate the social value of allowing trade in votes. As such, the message of the paper is generally positive, assuaging fears about “empty voting”. This result follows naturally from the endogeneity of vote supply because the natural suppliers of votes are unbiased, uninformed shareholders, and one would expect these shareholders to be unwilling to supply their votes at a zero price if doing so was expected to damage social welfare. Our result could be different, however, if votes are lent unwittingly or unknowingly — in which case there would be a role for policy makers to make institutions aware of their vote lending practices and to encourage them to recall shares around lending dates in case they wanted to vote those shares based on their own information.

28 The broker and custodian bank may fulfill this role in corporate share-lending markets.
29 This strategy may be less successful in corporate vote trading markets than in the political elections on which these authors focus, at least in the US, as US corporate law contains many provisions to protect minority shareholders from abuses by majority shareholders.
30 See Aggrawal et al (2012) for a discussion of the changing institutional attitude towards the recall of loaned shares around the record date for corporate elections.
We have also shown that policies to raise voter turnout can be inadvertently welfare-reducing. In the US, if shareholders do not vote their shares, their broker is authorized to do so under certain circumstances. We model this policy by assuming that if a voter chooses to abstain, his votes will with some probability be voted randomly. We show that this reduces social welfare (and the welfare of the voter) in three ways: first, it directly introduces more noise into the election, making it less likely that the voting outcome reflects the information of informed voters, and second, as a response to this, it makes the voter in question more likely to vote in favor of his bias; and third, other biased voters react by doing the same.

A limitation of our results is that we do not endogenize information acquisition by voters: we model voters as either informed or uninformed. In reality, if a voter knows that if he tries to abstain his vote will be randomly voted, this makes him more determined to vote; which may in turn provide him with greater incentives to collect information regarding the correct decision in the election when information collection is potentially costly. It is interesting, in this regard, to reflect on the increasing importance of the proxy voting advisory firms (ISS and Glass Lewis) as a response to the increased pressure on institutions to be seen to vote the shares that they hold (Li, 2013, and Aggrawal et al, 2012). The ability to lend votes to informed voters may reduce information acquisition by some voters; but the ability to pick up votes to improve voting power when informed may encourage information acquisition by others. We leave the study of this interesting trade off to future research on the interaction of endogenous information acquisition with endogenous vote supply.
Proof of Lemma 1. Recall that the probability that the net number of random votes cast with the state is \( n \) equals \( e^{-x} I_n(x) \). The probability that the decision matches the state is \( \sum_{n=K+1}^{\infty} e^{-x} I_n(x) \); we need to show that this quantity is increasing in \( x \) for all \( K \geq 1 \).

By property (iv) of Lemma A.1, we have \( I_n(x')/I_{n-1}(x') > I_n(x)/I_{n-1}(x) \) for all \( x' > x \), that is,

\[
\frac{e^x I_n(x')}{e^{x'} I_{n-1}(x')} > \frac{e^x I_n(x)}{e^{x'} I_{n-1}(x')}, \quad \forall x' > x.
\]  

(A.1)

At the same time, we have that \( e^{-x} I_0(x) + 2 \sum_{n=1}^{\infty} e^{-x} I_n(x) = 1 \) for any \( x \) because probability mass functions must sum to 1, so that

\[
\left[ e^{-x} I_0(x) - e^{-x'} I_0(x') \right] + 2 \sum_{n=1}^{\infty} \left[ e^{-x} I_n(x) - e^{-x'} I_n(x') \right] = 0.
\]  

(A.2)

(A.1) implies that if \( e^{-x} I_n(x) - e^{-x'} I_n(x') < 0 \) for some \( n \), then \( e^{-x} I_n(x) - e^{-x'} I_n(x') < 0 \) for all \( n' > n \). Hence we must have that \( e^{-x} I_0(x) - e^{-x'} I_0(x') > 0 \) since otherwise (A.2) would be violated. Similarly, by (A.2) there must also exist some \( n^* \) for which \( e^{-x} I_{n^*}(x) < e^{-x'} I_{n^*}(x') \). These observations, combined with (A.2), imply that

\[
2 \sum_{n=K}^{\infty} \left[ e^{-x} I_n(x) - e^{-x'} I_n(x') \right] < 0
\]

for all \( x' > x \) and \( K \geq 1 \), which is equivalent to what we wanted to prove. \( \square \)

Proof of Proposition 1. (i) Existence. Rewrite (4.1) as

\[
R_{K-1}(x(\tau^*)) \equiv \frac{I_K(x(\tau^*))}{I_{K-1}(x(\tau^*))} \leq \frac{1 - \tau}{1 + \tau},
\]  

(A.3)

where \( x(\tau^*) = \nu + \mu[1 - \tau^*/T] \) for \( \tau^* \in [0, T] \).
If $\tau^* = 0$ then (A.3) holds strictly for $\tau = 0$ because the Bessel-quotient is always less than 1, hence the abstention threshold must exceed 0. If $\tau^* = 1$ then (A.3) fails strictly for $\tau = 1$ so the threshold must be less than 1. By continuity there is a fixed point. Uniqueness follows because the Bessel-quotient on the left-hand-side of (A.3) is strictly concave in $x$ and $x^*(\tau)$ is linear, whereas $(1 - \tau)/(1 + \tau)$ is strictly convex.

(ii) Comparative statics. This can be seen by simply “shifting curves”.

On the left-hand side of (A.3) the Bessel-quotient is decreasing in $K$, hence if $K$ increases the inequality holds strictly at $\tau = \tau^*$, and so in order to restore equilibrium $\tau^*$ must rise.

As the expected number of noise voters or uninformed weak partisans increases (higher $\nu$ or $\mu$), $x(\tau^*)$ increases, and so does the Bessel-quotient on the left-hand side of (A.3). The inequality fails; in order to restore equilibrium $\tau^*$ must fall.

Finally, for a larger $T$ (a mean-preserving spread of the distribution of $\tau$), with the same threshold $\tau^*$ noise increases, the Bessel-ratio increases too, hence to maintain equilibrium $\tau^*$ must fall. □

Proof of Proposition 2. The equilibrium strategies are as follows. Each of the $\rho M$ uninformed shareholders that are called to the market submits a zero ask price (selling a single vote), $r_i(-1) = 0$. At the voting stage, if $\hat{K} - K$ votes have been traded in the vote market, then all $M$ uninformed shareholders use a threshold strategy $\tau^*$ such that with $x(\tau) = \nu + [1 - \pi(\tau)]\mu$,$$
(1 + \tau) [I_{\hat{K}}(x(\hat{\tau})) + I_{\hat{K}+1}(x(\hat{\tau}))] - (1 - \tau) [I_{\hat{K}}(x(\hat{\tau})) + I_{\hat{K}-1}(x(\hat{\tau}))] = 0.
$$

By Proposition 1 such $\hat{\tau}$ exists and is part of a threshold equilibrium of the voting subgame when $\hat{K}$ votes are to be cast for the state and $x(\hat{\tau})$ votes are expected to be cast at random. On the equilibrium path $\hat{K} = K + \rho M > K$, therefore by the comparative statics results established in Proposition 1 we have $\hat{\tau} > \tau^*$.

In the proposed equilibrium each informed shareholders submits $r_i(d_i) = 0$ for all $d_i \in \mathbb{Z}_+$ at the vote-trading stage. Therefore, on the equilibrium path, all $\rho M$ supplied votes are acquired by informed shareholders at zero price. There is no strategy that would induce a negative price for empty votes, and submitting a positive bid (potentially paying for an empty vote that would otherwise be acquired for free or by another informed shareholder) is suboptimal. Indeed it is a dominant strategy for informed shareholders to buy as many votes at zero price as they can and then to cast all their votes for the state.

Anticipating that votes bought at the vote-trading stage are cast by informed shareholders for the state, and not wanting to crowd out informed shareholders’ demands for empty votes, each uninformed shareholder strictly prefers not to bid but instead supply his or her vote in the vote-trading stage. □
REFERENCES


