Price Discrimination with Search*

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Abstract

We study price discrimination in a model of search. Two novel features of our model are that (i) buyers differ in size, and (ii) buyers endogenously choose whether to participate in search. Large buyers gain more from search. However, since the small buyers’ participation constraint is binding, conditional on search, small buyers are expected to have lower search costs. Since sellers compete more strongly for buyers with higher willingness to search, the relationship between buyers’ size and prices depends on which of these two countervailing effects dominates. We find that for sufficiently large buyers, size is always advantageous. For smaller buyers, size is (not) advantageous if the elasticity of the search cost distribution is decreasing (increasing); if it is iso-elastic, all small buyers get uniform prices. In this case, a ban on price discrimination would foster more participation by small buyers, unambiguously increasing overall welfare.

Keywords: price discrimination, search, online markets, OTC markets, electricity markets.

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1 Introduction

Price discrimination is a widespread phenomenon in many markets, from over-the-counter (OTC) markets for financial products and services (Jankowitsch et al., 2011; Reitz et al., 2015) to online retailing (Baye et al., 2004; Puller and Taylor, 2012; Schiller, 2014). Despite the differences across these markets, they have one feature in common: buyers have imperfect information about sellers’ prices, and so they have to incur search costs in order to find the cheapest supplier. For instance, in OTC markets, buyers and sellers have to contact each other to negotiate the terms of trade, a process which entails communication costs, information and processing costs, or delays. And while the internet has drastically reduced search frictions, online search is time consuming that comes at the cost of crowding out other activities (Wallsten, 2013). Furthermore, both markets constitute an ideal environment for price discrimination, as sellers can observe a wide range of consumer characteristics and buyers have limited arbitrage possibilities.

Despite this, the common assumptions made in the search literature leave little scope for price discrimination. Specifically, the search literature typically assumes that all consumers are ex-ante identical, i.e., they all have unit demands and equal valuations of the good, and either their search costs are also equal, or they are assumed to be drawn from a common distribution. In numerous papers of the search literature (including Varian, 1980; Burdett and Judd, 1983; Stahl, 1989; Janssen and Moraga-González, 2004 among others), equilibrium price dispersion arises as sellers strike a balance between setting low prices to attract buyers with low search costs versus setting high prices to extract more surplus from buyers with high search costs. Thus, even though each seller offers a single price, price dispersion effectively leads buyers with low search costs to pay lower expected prices. However, this should not be interpreted as price discrimination as price differences occur across sellers, not at the firm level (Stole, 2008).

In this paper, we explicitly incorporate observable differences across buyers to allow for price discrimination. Rather than introducing asymmetries in willingness to pay, as assumed in the price discrimination literature, we allow buyers to differ in their willingness to search, as assumed in the search literature. In practice, it is reasonable to suspect that certain practices used by sellers indeed serve to discriminate buyers according to their willingness to search. For instance, in online retailing, price differences arise depending on the route to the product (e.g. if the buyer accessed the website through a price-comparison engine), or depending on the buyer’s logged-in status.

The common assumption made in the search literature is that all buyers have the same gains

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1The "Nielsen State of the Media: The Social Media Report – Q3 2011" reports that "search" is among the 10 top online activities to which people devote more time to.

2There are several other cases in which price discrimination and search frictions coexist, e.g. supermarkets dealing with suppliers (Competition Commission, 2008), or banks selling loans to investors (Freixas and Rochet, 1997; Martin-Oliver et al., 2008).

3Furthermore, sellers are restricted to offering linear prices, thus not allowing them to elicit consumers’ private information through two-part tariffs or contract menus.

4There is evidence showing that consumers who are directed to Shoplet.com through a price-comparison website get cheaper prices on average (Mikians et al., 2012), consistently with these customers having observed the quotes of other online retailers. Similarly, there is evidence of differential pricing for consumers who are logged-in on Amazon.com (Mikians et al., 2013), consistent with Amazon expecting these customers to be less likely to search elsewhere.
from search, while their (possibly) heterogeneous search costs are not observed by sellers. Hence, even if buyers are *ex-post* heterogeneous, sellers expect them to have the same willingness to search. In contrast, we allow buyers’ gains from search to be heterogenous and observable. Still, since search costs are buyers’ private information, sellers’ information about buyers’ willingness to search is imperfect. This gives rise to equilibrium price dispersion - for similar reasons as in the papers cited above-, but it further results in price discrimination *at* the firm level as sellers compete more strongly for buyers with higher willingness to search.

Our model incorporates two novel features. First, in order to capture heterogenous gains from search, we allow buyers to differ in size. In particular, since search costs are fixed per price quote, large buyers stand to gain more from search. With search costs drawn from a common distribution, size heterogeneity introduces ex-ante differences in buyers’ willingness to search. Second, in contrast to most search models that take participation as given,\(^5\) we allow buyers to endogenously choose whether to participate in search (Moraga et al., 2014 is an exception).\(^6\) Since large buyers gain more from search, they are also expected to engage in search more often, i.e., for a wider range of search cost realizations.

These features create two countervailing effects. On the one hand, in expected terms, sellers expect larger buyers to gain more from search (we refer to this as the “gain-from-search effect”). On the other hand, sellers infer that small buyers who participate in search must have relatively lower search costs (we refer to this as the “participation effect”). Which of these two countervailing effects dominates determines whether sellers compete more or less aggressively to serve bigger buyers.

The relationship between prices and size depends critically on the range of search costs as well as on the shape of the search cost distribution. The range of search costs matters since it determines the set of buyers for whom the participation constraint is not binding. Since the gains from search are increasing in size, buyers have to be sufficiently large for them to be willing to search for all search costs realizations. For these buyers, only the “gain-from-search effect” is at play, so that prices are lower the bigger their size. The wider the range of search costs, the higher is the critical buyer’s size for which this result holds.\(^7\)

For the remaining buyers, i.e., those who search only if their realized search cost if sufficiently low, the effect of size on prices depends on the shape of the search cost distribution. More specifically, the relative importance of the “gains-from-search effect” versus the “participation effect” depends on the elasticity of the search cost distribution. When this elasticity is decreasing (increasing), a bigger (smaller) buyer is expected to search more, which in turn leads sellers to compete more aggressively to serve them. Intuitively, when the search cost distribution is more elastic at lower values of the search cost, an increase in size affects more the probability that the buyer asks for a second quote (“gains-from-search effect”) than the probability that he asks for a first quote (\(^5\)The majority of search models assume that buyers search at least once e.g. under the assumption that search costs are sufficiently low, or that buyers can obtain the first quote for free.\(^6\)However, our modelling assumptions are very different from those of Moraga et al. (2014). In particular, they use a model of sequential search with differentiated products and no price discrimination.\(^7\)Indeed, when the search cost distribution is unbounded from above, this result does not hold for any consumer.)
“participation effect”). In turn, this implies that sellers compete more aggressively to serve bigger buyers. For iso-elastic functions, e.g. the uniform distribution, the two effects cancel out so that prices are constant in size.

Our results support the hypothesis that larger buyers have buyer power -a relevant concept for antitrust-, but only for very large buyers. For smaller buyers, size is advantageous only under certain assumptions on the shape of the search cost distribution. In line with existing evidence (Sorensen, 2000; Ellison and Snyder, 2010; Grennan, 2013; and the UK Competition Commission 2008’s inquiry into the groceries market), this suggests that the relationship between buyer’s size and prices is industry-specific. In comparison to the bilateral contracting literature, our result does not depend on the distribution of bargaining power between buyers and sellers, nor on the impact of buyer’s size on the parties’ outside option (Katz (1987); and Inderst and Montez (2014)).

An increase in switching costs has distinct effects on the various buyers. Whereas large buyers always benefit from a reduction in search costs, the effect on the small-sized buyers depends on the shape of the search cost distribution. We find conditions under which the prices for the small buyers increase even when search frictions become milder. Intuitively, small buyers are more likely to participate when search costs go down, thus making their participation decisions less informative of their search costs. If this second effect dominates, sellers will compete less aggressively to serve them.

Last, we derive results regarding the effects of banning price discrimination. When firms are forced to charge a uniform price to all consumers, firms price as if they were facing a consumer of average size. Thus, a ban on price discrimination has distinct effects across consumers. For instance, with constant elasticity, banning price discrimination reduces the prices for small buyers but increases those for the large ones. This induces the small buyers to engage in search more often, so that total consumption goes up. Since for the large buyers the participation constraint is non-binding, increasing their prices does not reduce their consumption. Hence, in this case, a ban on price discrimination has unambiguous effects: it increases overall welfare, making consumers better-off in the aggregate despite the detrimental effect on the large ones. These results are in line with those in (Armstrong and Vickers, 2001), about the effects of price discrimination in a Hotelling model for the case of observable heterogeneity across consumers.

The reminder of the paper is structured as follows. In section 2 we construct and solve the model; we first characterize sellers’ pricing decisions and buyers’ search behavior to then prove existence and uniqueness of the equilibrium. In section 3 we perform comparative statics with respect to buyers’ size and look at the effects of reducing search costs. In section 4 we assess the effects of banning price discrimination. Section 5 concludes.

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8For instance, in Katz (1987), buyers’ outside option is to procure the good from an alternative supplier. Large buyers’ stand at a stronger bargaining position since accessing the alternative supplier entails a fixed cost. In Inderst and Montez (2014), the buyer’s outside option is to relocate demand across various sellers. This creates countervailing effects as large buyers find it difficult to relocate all their demand to alternative suppliers, while sellers find it difficult to fill their capacity if the negotiations with large buyers do not go through. These forces imply that size provides a competitive advantage for buyers only when their bargaining power is sufficiently high.
2 The Model

Consider a market in which two symmetric sellers compete for selling an homogeneous good. They face constant returns to scale, with marginal costs normalized to zero. Each buyer has an inelastic demand for \( q \) units of the good. We refer to \( q \) as the buyer’s size, and allow it to differ across buyers. Buyers’ sizes are distributed according to the function \( H(q) \). Last, all buyers value the good at \( v \), which can be interpreted as either the gross surplus or the default price for those buyers who do not search.

In order to know sellers’ prices, buyers have to pay a fixed search cost \( c \) per price offer, including the first. Buyers’ search costs are private information, but it is common knowledge that they are identically and independently distributed according to \( G(c) \) in \([0, \bar{c}]\), with \( G'(c) > 0 \) and \( G''(c) \leq 0 \).

The timing of the game is as follows. First, buyers observe their realized search costs. In order to maximize their expected utility, they decide how many price quotes to ask for. Accordingly, the strategy of a buyer with search cost \( c \) and demand \( q \) is a number in \( \{0, 1, 2\} \), representing the number of price quotes he asks for. If the buyer has decided to ask for one quote only, both sellers are equally likely to receive the quote request. Sellers do not observe buyers’ search costs nor search decisions. Upon receiving a quote request for \( q \) units, each seller chooses his quote according to the distribution function \( F(p; q) \). Since sellers observe the buyer’s size, their offers can be made conditional on \( q \). Last, buyers always buy the good at the lowest quote among the ones they observe.

We examine (symmetric) rational expectations equilibria, i.e., buyers maximize their utility given their correct beliefs about the distribution of price offers in the market, and sellers maximize their profits given their correct beliefs about buyers’ search behavior. When price discrimination is allowed, and because of constant returns to scale, the offers made by sellers to a given buyer do not depend on the transactions with other buyers. Therefore, without loss of generality, we can focus on a single buyer. The distribution of buyers will become relevant in section 4 when we look at the case of uniform prices.

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9 We assume that sellers have constant returns to scale for our results not to depend on whether sellers face different costs when serving consumers of different size.

10 If applied to OTC markets, the implicit assumption is that both sellers have access to a wholesale market in which they can buy the good at the same price. If they produce the good themselves, they can always sell it in the wholesale market. Hence, regardless of their own production costs, which could be asymmetric, the wholesale market price is their opportunity cost.

11 We could introduce scale economies (diseconomies) in search by assuming that the second quote costs \( \delta c \), with \( \delta < 1 (\delta > 1) \). Our main results would remain qualitatively unchanged.

12 Since all firms are ex-ante identical, the consumer is indifferent about picking one firm or another. Our qualitative results would not change if we introduced asymmetries in this respect, i.e., if sellers always approached one of the two sellers first.

13 In the standard non-sequential search model, buyers and sellers play a simultaneous moves game. In this model, sellers quote prices upon receiving a quote request. This difference is relevant given that it allows the buyer’s participation decision to be informative about his search cost.

14 In section 4 we consider the case of uniform pricing, so that offers cannot be made contingent on \( q \).
2.1 Pricing decisions

Consider a seller who has received a quote request to supply quantity \( q \) to a given buyer. At this stage, it does not know how many quotes the buyer has asked for, but forms the following expectation: it believes that the buyer has asked for only one quote with probability \( \rho(q) \). Hence, the seller believes that the buyer has asked for a second quote with probability \( 1 - \rho(q) \). In equilibrium, this expectation must turn out to be correct. To simplify notation, in what follows we will simply write \( \rho \).

Our first result shows that in equilibrium, conditional on searching, some buyers search for one quote only while others search for two. Hence, the seller who has received a quote request cannot infer with certainty whether the buyer has asked for a second quote. The only information the seller can infer is that the buyer’s search cost is such that he is better off asking for at least one.

**Lemma 1** In a SPNE, if the buyer participates, \( \rho \in (0, 1) \).

The intuition for this result is well known. On the one hand, if the buyer asks for only one quote with certainty \( (\rho = 1) \), the seller charges the monopoly price. However, as this would leave no surplus for the buyer, he would not search in the first place. On the other hand, if the buyer asks for two quotes with certainty \( (\rho = 0) \), sellers engage in Bertrand competition. Since all sellers would then price at marginal costs, the buyer would have incentives to ask for one quote only to save search costs. As a consequence, neither the monopoly price nor the competitive price can be sustained in a SPNE.\(^{15}\)

More generally, \( \rho \in (0, 1) \) rules out the existence of pure strategy equilibria: starting from any arbitrary price pair, sellers would like to undercut each other until prices are so low that it becomes optimal for a seller to price at the buyer’s reservation price, \( v \). However, if one seller is pricing at \( v \), it becomes profitable for a seller to price slightly below that level, and undercutting becomes optimal again. Therefore, the equilibrium has to be in mixed strategies.

When asked to provide a quote for quantity \( q \), let sellers use the (symmetric) quote distribution \( F(p; q) \). A seller’s profits from quoting \( p \) are given by,

\[
\pi(p; q) = pq [\rho + (1 - \rho) (1 - F(p; q))].
\]

A seller’s mixed strategy strikes a balance between the benefits of charging a high price when the buyer has asked for only one quote, an event which occurs with probability \( \rho \), versus the benefits from being the low-priced seller when the buyer has asked for two quotes, an event which occurs with probability \( (1 - \rho) (1 - F(p; q)) \). Proposition 1 below characterizes the equilibrium quote distribution.

\(^{15}\)If the first quote is free, as in Burdett and Judd (1983), there always exists an equilibrium where all firms charge the monopoly price.
Proposition 1 Assume $\rho \in (0, 1)$. There is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by

$$F(p; q) = 1 - \frac{\rho(v - p)}{1 - \rho}$$

with compact support $p \in [v\rho, v]$.

Buyers’ search behavior affects the quote distribution through $\rho$. An increase in $\rho$ shifts the whole quote distribution downwards in a FOSD sense, also reducing the lower bound of the price support. Therefore, an increase in $\rho$ implies that sellers price less aggressively, leading to higher expected quotes. The fact that $\rho$ shrinks the quote support does not mean that dispersion is reduced because $\rho$ also affects the shape of the distribution. Indeed, an increase in $\rho$ has a non-monotonic effect on the dispersion of quotes: it first increases and it then decreases as $\rho$ goes up.

Buyers buy from the seller offering the lowest quote among the ones they observe. If the buyer has asked for one quote (which occurs with probability $\rho$), he will simply pay that quote. However, if he has asked for two quotes (which occurs with probability $1 - \rho$), he will accept the minimum of the two. Accordingly, using the quote distribution characterized in Proposition 1, one can characterize the distribution of prices, which we can use to compute expected prices and price dispersion for given $\rho$. Conditional on searching, expected prices are given by

$$\rho E[p] + (1 - \rho) E[\min\{p_1, p_2\}]$$

where

$$E[p] = \int_{v\rho}^{v} p dF(p; q) > \int_{v\rho}^{v} 2p (1 - F(p)) dF(p; q) = E[\min\{p_1, p_2\}]$$

An increase in $\rho$ leads to higher expected prices. for a two-fold reason: first, the higher $\rho$, the less likely it is that the buyer can compare two quotes to choose the lowest one; and second, the higher $\rho$, the weaker is the competition between the sellers. In contrast, an increase in $\rho$ has a non-monotonic effect on price dispersion. This is summarized in the following Lemma:

Lemma 2 For all $\rho \in (0, 1)$,

(i) The equilibrium price distribution is decreasing in $\rho$.

(ii) Expected prices are increasing in $\rho$.

(iii) There exists $\hat{\rho}$ such that the variance of prices is increasing in $\rho$ if and only if $\rho \in (0, \hat{\rho})$.

While the above Lemma provides comparative statics with respect to $\rho$, we are ultimately interested in the comparative statics with respect to quantity $q$. The fact that $q$ does not enter directly into the quote distribution does not mean that sellers choose equal quotes to all buyers, and hence charge equal expected prices on average, regardless of their size. Indeed, as we analyze next, $q$ enters $F(p; q)$ indirectly through the value of $\rho$, as buyers’ equilibrium search intensity depends on $q$. 

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2.2 Search decisions

Conditionally on searching, the buyer chooses how many price quotes to request and then buys from the seller quoting the lowest price in his sample. The utility of a buyer who has asked for either one or two quotes, when he is demanding $q$ units and his search cost is $c$, is given as follows

\[ u_1(q, c) = (v - E[p]) q - c \]
\[ u_2(q, c) = (v - E[\min\{p_1, p_2\}]) q - 2c. \]

The buyer will engage in search if his utility from search is non-negative, i.e., if his expected surplus from search exceeds his search cost. Conditional on search, the buyer trades-off the likelihood of observing a lower price versus the search cost of getting an additional quote. Thus, the buyer will ask for two quotes if the expected price savings exceed his search cost. Accordingly, the buyer’s optimal search behavior follows a cut-off strategy, which is characterized next:

**Proposition 2** For given $\rho \in (0, 1)$, there exist $v > c_0 > c_1 > 0$ such that a buyer of size $q$

(i) does not search if $c > c_0$, asks for one quote if $c \in (c_1, c_0]$, and asks for two quotes if $c \leq c_1$.

(ii) Furthermore, $c_0$ and $c_1$ are linear in $q$; $c_0$ is decreasing in $\rho$, while $c_1$ is non-monotonic in $\rho$.

For a buyer to find it optimal to engage in search, his search cost has to be sufficiently low,

\[ c \leq c_0 = (v - E[p]) q \]

His search cost has to be even smaller for him to find it optimal to ask for two quotes,

\[ c \leq c_1 = (E[p] - E[\min\{p_1, p_2\}]) q \]

Importantly, the thresholds that characterize optimal search, $c_0$ and $c_1$, are increasing in $q$. Intuitively, larger buyers stand to gain more from search given that the potential benefits are proportional to the quantity demanded. Hence, in expected terms, larger buyers are more likely to engage in search and to ask for two quotes.

However, sellers make their offers once buyers have decided to engage in search. Hence, from their point of view, what matters is buyers’ willingness to search *conditional on search*, and this depends both on the buyers’ gains as well as on their costs. As already argued, the gains from search are known to be higher for bigger buyers. We refer to this as the “gains-from-search effect”. However, sellers expect smaller buyer to have lower search costs given that participation decisions truncate the search cost distribution at $c_0$, which is increasing in $q$. We refer to this as the “participation effect”. Thus, whether sellers expect large or small buyers to be more likely to ask for two quotes will critically depend on the interplay between these two countervailing effects (section 3.1).
Moreover, the thresholds $c_0$ and $c_1$ are a function of $\rho$ given that $\rho$ affects expected prices. Specifically, $c_0$ is strictly decreasing in $\rho$ since expected prices are increasing in $\rho$ (Lemma 2), thus making the participation condition more stringent. On one extreme, $c_0 = vq$ if $\rho = 0$: since prices are equal to (zero) marginal costs, the buyer searches whenever his search cost $c$ is below his gross surplus $vq$. On the other extreme, $c_0 = 0$ if $\rho = 1$: since prices are equal to the buyer’s reservation value, he never engages in search, regardless of his search cost realization. In contrast, $c_1$ is non-monotonic in $\rho$ given that $\rho$ affects expected prices both when the buyer asks for one quote as when he asks for two. These features turn out to be very important for the equilibrium characterization, as discussed next.

### 2.3 Equilibrium characterization

Since sellers’ expectations must be fulfilled in equilibrium, the probability of the buyer asking for one quote, conditional on search, must satisfy

$$
\rho = 1 - G(c_1|c \leq c_0) = 1 - \frac{G(c_1)}{G(c_0)} \in (0, 1).
$$

If the highest search cost realization $\bar{c}$ is below $c_0$, by Proposition 2, the buyer engages in search for all $c$. Hence, $G(c_0) = 1$. Otherwise, if $\bar{c}$ is above $c_0$, the buyer does not search for $c$ in $[c_0, \bar{c}]$ so that $G(c_0) < 1$. Hence, equation (1) can be re-written as

$$
\rho = \begin{cases} 
1 - G(c_1) & \text{if } \bar{c} \leq c_0 \\
1 - \frac{G(c_1)}{G(c_0)} & \text{if } \bar{c} \geq c_0
\end{cases}
$$

Furthermore, since $c_0$ is strictly decreasing in $\rho$, (for given $q$) there exists a unique value $\bar{\rho} \in [0, 1]$ such that $\bar{c} \geq c_0$ if and only if $\rho \geq \bar{\rho}$. Hence, equation (1) can be re-written as

$$
\rho = \begin{cases} 
1 - G(c_1) & \text{if } \rho < \bar{\rho} \\
1 - \frac{G(c_1)}{G(c_0)} & \text{if } \rho \geq \bar{\rho}
\end{cases}
$$

Intuitively, for $\rho$ below $\bar{\rho}$, expected prices are low, and the buyer always finds it profitable to search. Therefore, the buyer’s participation decision is not informative about his search cost. As $\rho$ increases above $\bar{\rho}$, expected prices go up, and the buyer only finds it optimal to search for low search cost realizations. In this case, the buyer’s signals that his search is below $c_0$.

Figure 1 plots the right hand side of the equilibrium condition (1) as a function of $\rho$, for low, medium and high $q$. The schedules have two regions: to the left of the kink, the figures plot $1 - G(c_1)$, to the right of the kink, the figures plot $1 - \frac{G(c_1)}{G(c_0)}$. The schedules shift down as $q$ goes

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16 In order to simplify notation, we don’t make it explicit that $c_0$ and $c_1$ depend on $\rho$ unless needed for clarity.

17 In particular, $c_1$ first increases and then decreases in $\rho$. Furthermore, $\lim_{\rho \to 0} c_1 = \lim_{\rho \to 1} c_1 = 0$.

18 Note that $\bar{\rho}$ is a function of $\frac{v}{q}$. In turn, since $c_0$ is strictly decreasing in $\rho$, then $\bar{\rho}$ is increasing in $q$.

19 The figures have been depicted under the assumption of uniformly distributed search costs in $[0, 1]$ and $v = 5$; low $q = 1$; medium $q = 1.2$ and high $q = 1.5$.

20 Note that the function $1 - G(c_1)$ inherits the non-monotonicity of $c_1$. 

up: because of the gains-from search effect, a larger buyer is less likely to ask for one quote only. Similarly, the kink moves to the right as \( q \) goes up: there is a wider range of \( \rho \) values for which a larger buyer always searches. The solution to equation (1) is given by the intersection of the schedule with the 45-degree line. In Figure 1, when \( q \) is low (high), the equilibrium lies to the right (left) of the kink; for medium \( q \), the solution lies at kink.

More generally, the following Theorem guarantees that there always exists a solution to equation (1), which is interior.\(^ {21} \) Together with our previous results, the solution to (1) completes the characterization of the SPNE.

**Theorem 1** There exists a symmetric SPNE in which sellers price as stated in Proposition 1 and buyers search as stated in Proposition 2. Conditional on participating, the probability that the buyer asks for one quote is given by the solution to (1). If \( G \) satisfies the MLRP, the solution is unique.

With a slight abuse of notation, we write \( c_0^* \) to denote \( c_0 \) in equilibrium.\(^ {22} \)

**Proposition 3** \( c_0^* \geq \bar{c} \) if and only if \( q \geq \bar{q} = 6/vG''(0) \).

The buyer who demands \( \bar{q} \) is the smaller buyer for which the participation constraint does not bind. For buyers with \( q > \bar{q} \), the participation condition never binds.\(^ {23} \) Thus, the equilibrium is the solution to \( \rho^* = 1 - G(c_0^*) \). In contrast, for buyers with \( q < \bar{q} \), the participation condition binds for some search cost realizations. In this case, the equilibrium condition is \( \rho^* = 1 - \frac{G(c_0^*)}{G(c_0^*)} \).

\(^{21}\)Unless we impose conditions on \( G \), we cannot guarantee that the solution is unique.

\(^{22}\)In equilibrium, \( c_0^* \) depends on \( q \) through \( \rho \).

\(^{23}\)Note that this case never arises when the search cost distribution is unbounded by above.
In what follows, we refer to buyers with \( q \geq \bar{q} \) as large and buyer with \( q < \bar{q} \) as small buyers. This classification between large and small buyers is endogenous given that \( c_0^* \) depends on \( \rho \), which is determined in equilibrium (1). Thus, changes in parameters affect the threshold \( \bar{q} \).

## 3 Equilibrium Comparative Statics

How do sellers use the information about buyers’ expected willingness to search to price discriminate among them? How do changes in the search cost distribution affect the prices paid by buyers of different size? In this section we shed light on these questions. In section 3.1 we perform comparative statics with respect to buyers’ size, while keeping the search cost distribution constant, while in 3.2 we keep sizes unchanged while performing comparative statics with respect to the search cost distribution.

### 3.1 Buyers’ size

Since we are interested in understanding the link between price discrimination and buyers’ size, we now perform equilibrium comparative statics with respect to \( q \).

**Proposition 4** The relationship between expected prices and buyers’ size is given as follows:

(i) (Large buyers) If \( q \geq \bar{q} \), expected prices are unambiguously decreasing in \( q \).

(ii) (Small buyers) If \( q < \bar{q} \), expected prices are (strictly) decreasing in \( q \) if and only if

\[
\varepsilon(c_1) > \varepsilon(c_0),
\]

where \( \varepsilon(c) \equiv \frac{\partial G(c)}{\partial c} \) is the elasticity of the search cost distribution. A sufficient condition for this is that \( \varepsilon'(c) < 0 \) for all \( c \). For distribution functions \( G(c) \) with constant \( \varepsilon(c) \), expected prices do not depend on \( q \).

Since large buyers always engage in search, their participation decisions are not informative about their search costs. Since only the “gains-from-search effect” is at play, prices decrease in \( q \) over this range. This holds true regardless of the shape of the search cost distribution function \( G(c) \).

To the contrary, for small buyers, the “gains-from-search effect” and the “participation effect” work in opposite directions. Whether one effect or the other dominates, leading to prices which are either decreasing or increasing in \( q \), depends on the shape of \( G \). In particular, it depends on the elasticity of the search cost distribution. Suppose that the elasticity of \( G(c) \) is decreasing in \( c \). Since \( c_1 < c_0 \), this implies that the elasticity is higher at \( c_1 \) than at \( c_0 \). Hence, an increase in \( q \) increases the probability that the buyer asks for two quotes, \( G(c_1) \), more than it increases the probability that the buyer participates, \( G(c_0) \). Since this implies that the “gains-from-search effect” outweighs the “participation effect”, an increase in \( q \) leads to lower expected prices. The contrary is true if the elasticity is increasing in \( c \).²⁴ Last, for \( \varepsilon(c) \) constant, the “gains-from-search”

²⁴It is important to note that even when small buyers get lower prices than large ones, small buyers need not be better off as compared to large ones, given that the probability the small ones engage in search is also smaller.
and the “participation” effects cancel out. Hence, all buyers pay the same price, regardless of their size. In other words, sellers find it optimal not to price discriminate. This result illustrates the importance of endogenizing participation decisions: assuming that all buyers search at least once would mute the “participation effect”, and size would unambiguously be pro-competitive.

There are several commonly used distribution functions for which \( \varepsilon (c) \) is everywhere decreasing. For instance, the exponential function, the logistic function or the standard normal. There are also other distribution functions for which \( \varepsilon (c) \) is increasing. The family of functions \( G(c) = c^\varepsilon \), with \( \varepsilon \leq 1 \), including the uniform distribution, have constant elasticity.

### 3.2 Search costs

We want to understand how a reduction in search costs affects equilibrium pricing and how the effects differ across buyers of different sizes. For this purpose, we perform equilibrium comparative statics with respect to search costs. Let us parametrize the search cost distribution as \( G(c; \lambda) \), where an increase in \( \lambda \) shifts the distribution function upwards in a First Order Stochastic Dominance, i.e. \( \partial G(c; \lambda) / \partial \lambda > 0 \). Hence, an increase in \( \lambda \) is equivalent to a reduction in search costs.

The following Proposition summarizes the effect on prices of a marginal reduction in search costs, as parametrized by \( \lambda \). Since the threshold \( \bar{q} \) might depend on \( \lambda \), we write \( \bar{q}(\lambda) \).

**Proposition 5** The effects on equilibrium prices of a marginal reduction in search costs are as follows:

(i) **(Large buyers)** If \( q \geq \bar{q}(\lambda) \), expected prices are decreasing in \( \lambda \).

(ii) **(Small buyers)** If \( q < \bar{q}(\lambda) \), expected prices are decreasing in \( \lambda \) if and only if the elasticity with respect to \( \lambda \), \( \varepsilon (c; \lambda) \equiv \frac{\partial G(c; \lambda)}{\partial \lambda} \frac{\lambda}{G(c; \lambda)} \), is higher at \( c_1 \) than at \( c_0 \). A sufficient condition is that \( \varepsilon (c; \lambda) \) is everywhere decreasing in \( c \). For distribution functions \( G(c; \lambda) \) with constant elasticity, expected prices do not depend on \( \lambda \).

A reduction in search costs creates two countervailing effects. First, conditional on participating, buyers are more likely to ask for a second quote; hence, sellers price more aggressively. However, since asking for a first quote is also less costly, participation decisions are less informative of the realized search cost. In particular, sellers expect those buyers for whom the participation condition is binding to have relatively higher search costs; hence, sellers price less aggressively to serve them.

Large buyers always engage in search. Hence, only the first effect is at play, and lower search costs unambiguously lead them to pay lower prices. For the small buyers, since the participation condition is binding, the two effects are at play. Whether the first or the second effect dominates depends on whether the percentage reduction in search costs affects more the upper deciles of the search cost distribution (which affect the probability of a second quote) or the lower ones (which affect participation decisions). In particular, if the percentage increase in \( G(c_1; \lambda) \) is stronger...

\[ \text{\scriptsize 25} \] Alternatively, one could parametrize it as a function of \( \beta \) where \( \lambda = 1/\beta \). In this case, \( \partial G(c; \beta) / \partial \beta > 0 \) so that an increase in \( \beta \) would be equivalent to an increase in search costs.
(weaker) than that in \( G(c_0; \lambda) \), then a reduction in search costs leads small buyers to pay lower (higher) prices. If the reduction in search costs affects all deciles uniformly, then the prices paid by small buyers remain constant despite the reduction in search costs. This is always the case whenever \( \lambda \) enters the search cost distribution multiplicatively, i.e., \( G(c; \lambda) = \lambda G(c) \).

When we consider marginal changes in \( \lambda \), the effect on the buyer demanding \( q \) is nil because the pattern of expected prices is continuous in \( q \). However, this is no longer the case if we compare prices across two search costs distributions, i.e., \( G(c; \lambda') \) and \( G(c; \lambda'') \) with \( \lambda' < \lambda'' \). As shown next, the critical size \( q(\lambda) \) is decreasing in \( \lambda \), thus implying that some of the “small” buyers under \( \lambda' \) become “large” under \( \lambda'' \). Intuitively, when search costs go down, the participation condition for these buyers is no longer binding for any search cost. Hence, for buyers with \( q \) in \([\overline{q}(\lambda''), \overline{q}(\lambda')]\), the reduction in search costs is pro-competitive. This is summarized below:

**Lemma 3** For any \( \lambda' < \lambda'' \), \( \overline{q}(\lambda'') < \overline{q}(\lambda') \).

In sum, under certain conditions, a reduction in search costs reduces the prices paid by large buyers only, and can even lead to price increases for the small ones. Large buyers are unambiguously better off both because they pay lower prices and incur lower expected search costs. The effect on the welfare of small buyers is ambiguous: on the one hand, lower search costs allow them to engage in search more often and thus benefit from greater consumption; however, they might also end up paying higher prices.

### 4 Uniform Prices versus Price Discrimination

We now turn our attention to the effects of banning price discrimination. In this section we characterize pricing and search decisions in equilibrium when sellers are forced to charge equal prices to all consumers.\(^{26}\)

For similar reasons as in the price discrimination case, there does not exist a pure-strategy equilibrium; in particular, sellers do not know with certainty whether buyers have asked for only one or for more than quote. Hence, when asked to provide a quote, sellers use the (symmetric) quote distribution \( F(p) \), which can no longer depend on \( q \). A seller’s profits from quoting \( p \) are given by,

\[
\pi(p) = p \int q \left[ \rho(q) + (1 - \rho(q)) (1 - F(p)) \right] dH(q)
\]

where \( \rho(q) \) is the probability with which a buyer with size \( q \) asks for only one quote.

**Proposition 6** There is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by

\[
F(p) = 1 - \frac{\tilde{\rho} \cdot v - p}{1 - \tilde{\rho} \cdot p}
\]

with compact support \( p \in [\underline{p}, v] \) where \( \tilde{\rho} = \frac{\int q \rho(q)dH(q)}{\int q dH(q)} < 1 \).

\(^{26}\)The analysis would be the same if we assumed that sellers do not observe buyers’ size.
This equilibrium is analogous to the one under price discrimination (Proposition 1), with the difference being that $\bar{\rho}$ now reflects the weighted average of the different values of $\rho$ across the population of buyers. Hence, those consumers with $\rho(q)$ above (below) the average face are charged lower (higher) prices when price discrimination is banned.\footnote{Interestingly, note that the distribution of sizes (which is exogenous) affects $\bar{\rho}$ and hence the prices charged to all buyers in the no-discrimination case. In this sense, changes in the distribution of buyers’ size (e.g. following a merger between downstream retailers) affect the degree of competition among sellers, thus affecting all buyers in the industry. The effect might be positive or negative depending on whether the average $\bar{\rho}$ decreases or increases after the merger, an issue which will depend on the relative sizes of the merging parties.}

For a given pricing behavior of the sellers, buyers’ search decisions do not depend on whether price discrimination is allowed or not. However, search behavior will ultimately be affected to the extent that sellers’ pricing behavior changes depend on whether price discrimination is allowed or not. Accordingly, Proposition 2 remains unchanged, with the sole difference that the equilibrium values of $\rho(q)$ are now found as the solution to the following system of equations:

$$\rho(q) = 1 - G(c_1(\bar{\rho}; q)| c \leq c_0(\bar{\rho}; q)),$$  for all $q$

Each of these conditions is analogous to (1), with the difference being that the right hand side now depends on the average $\bar{\rho}$.

A ban on price discrimination affects participation decisions, and through that, it has an effect on overall welfare. The following Proposition demonstrates that in the case in which the search cost distribution has constant elasticity, a ban on price discrimination would unambiguously increase welfare.\footnote{Our implicit assumption is that if consumers do not search, they do not consume the good.}

Indeed, the prices for all consumers for whom the participation constraint is binding (those with $q \leq \bar{q}$) would go down. Furthermore, this would induce them to engage in search for a wider range of search cost realizations. The large consumers are split into two groups, $q \in (\bar{q}, \bar{q})$ and $q > \bar{q}$. The prices for those consumers with $q > \bar{q}$ would go up; this would not induce them to engage less in search given that their participation constraint is satisfied with slack.\footnote{Indeed, we can be sure that their participation constraint is still satisfied with slack after the ban on price discrimination: they have higher gains from search than consumers with $q \in (\bar{q}, \bar{q})$, for whom the participation constraint is satisfied with slack before and after the ban.}

The remaining group of consumers, those with $q \in (\bar{q}, \bar{q})$, keep on engaging in search for all search cost realizations while paying lower prices. Hence, all consumers with $q < \bar{q}$ benefit from a ban on price discrimination, while consumers with $q > \bar{q}$ are worse off. Overall, there is an increase total surplus because of the effects on participation (with demands being price inelastic, the prices effects imply a pure transfer between buyers and sellers). This is summarized next:

**Proposition 7** If $\varepsilon'(c) = 0$, a ban on price discrimination increases overall welfare. Furthermore, there exists $\bar{q} > \bar{q}$ such that a consumers with size $q < \bar{q}$ are better off and the remaining consumers are worse off.
consumers engage in search less often after the ban on price discrimination.

5 Conclusions

In this paper we have analyzed the interaction between price discrimination and imperfect competition in markets with search costs. Buyers heterogeneity regarding their willingness to search opens the scope for price discrimination. In particular, sellers expect buyers with a high willingness to search to search more, and hence, they compete more aggressively to serve them.

Even though sellers do not observe buyers’ willingness to search, they obtain some imperfect signals. Buyers’ willingness to search can be decomposed into the gains from search and the costs of search. In our model, the gains from search depend on the buyer’s size, which is observable; the costs of search depend on the buyer’s realized cost, which is non-observable. Since large buyers have more to gain from search, and search costs are drawn from a common distribution, sellers expect large buyers to search more in expected terms. However, buyers’ participation decisions are also informative of their search costs as these have to be sufficiently low for search to be optimal. Since small buyers engage in search for a narrower range of search costs, sellers expect the search costs of the small buyers to be lower. Thus, whether large or small buyers are expected to have higher or lower willingness to search depends of the strength of these two countervailing effects. This issue ultimately depends on the shape of the search cost distribution function. We identify properties of it under which large buyers pay less, the same or more as compared to small buyers.

Our analysis provides an unambiguous prediction regarding the effects of price-discrimination when search costs are uniformly distributed (or more broadly, when the search cost has constant elasticity): price discrimination reduces overall welfare. The reason is that it increases prices for some consumers for whom the participation constraint is binding, while it reduces prices for consumers who would search in any event. Since this reduces the amount of consumers who engage in search, overall consumption is reduced.

These issues are becoming increasingly relevant for regulators and competition policy authorities as the internet, or the IT technologies more broadly, provide a vast source of data on consumers characteristics that sellers can use to price discriminate in a wide range of markets.\textsuperscript{30} The question of whether a ban on price discrimination would be welfare enhancing is still open to a lively debate.

References


\textsuperscript{30}Fort instance, in January 2015, the UK Competition and Markets Authority has open a call for information on the commercial use of consumer data. The European Commission has also raised concerns about residence-based price discrimination in several occasions. In March 2015, the EC has launched a competition inquiry into the e-commerce sector, with price-discrimination as one of its implicit concerns.


Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1. Suppose $\rho = 1$. Then, the seller knows that it is a monopolist and hence charges the reservation price, $v$. However, it would leave no surplus for the buyer, so his participation constraint would not be satisfied. Hence, $\rho = 0$ cannot be part of an equilibrium in which the buyer has decided to participate. Suppose $\rho = 0$. Then, there is Bertrand competition with both sellers quoting prices equal to marginal cost. However, knowing that all sellers choose the same quote, the buyer would have incentives to deviate and ask for fewer quotes in order to save on search costs. Hence, $\rho = 0$ cannot be part of an equilibrium. As a consequence, in equilibrium, we must have $\rho \in (0,1)$.

Proof of Proposition 1. Let sellers choose the distribution of quotes $F(p)$ over the interval $[\bar{p}, \overline{p}]$. Standard arguments imply that this distribution must be atomless. In particular, if sellers put mass at a given quote, there would be a positive probability of a tie. If such a quote is above marginal costs, each seller would be better off by slightly reducing its quote below that level: this would have a minor effect on its profits if the buyer has asked for one quote only, but would guarantee that the seller makes the sale whenever the buyer has asked for two quotes. Putting mass at marginal cost cannot be part of an equilibrium either, given that each seller would be able to make the sale at a higher price whenever the buyer has asked for one quote only.

Seller $i$’s profits from quoting a price $p$ when rivals are choosing quotes according to $F(p;q)$ are given by:

$$
\pi (p; q) = pq \left( \rho + (1 - \rho) \left( 1 - F(p; q) \right) \right).
$$
Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(p; q) = \pi$$ for all $p \in [\underline{p}, \overline{p}]$.

Furthermore, as $\pi(\overline{p}; q) = \overline{p} q \rho$ is increasing in $\overline{p}$, it follows that $\overline{p} = v$. Hence, $\pi = v q \rho$. From $\pi(\overline{p}; q) = p q = \pi = v q \rho$ it also follows that $\overline{p} = v \rho$. Accordingly, the support of the equilibrium mixed strategy is $p \in [v \rho, v]$.

To obtain the equilibrium quote distribution, from $\pi(p) = v q \rho$, it follows that

$$F(p; q) = 1 - \rho \frac{v - p}{1 - \rho}$$ for $p \in [v \rho, v]$$

with density

$$f(p; q) = \rho \frac{v}{1 - \rho}$$

For $\rho \to 1$, the equilibrium collapses to the reservation price, whereas for $\rho \to 0$, the equilibrium collapses to marginal costs.

Sellers’ expected profits are thus

$$\pi(p; q) = v q \rho.$$

Integrating over all buyers,

$$\pi(p) = \int \pi(p; q) dH(q) = v \int q \rho(q) dH(q)$$

where as have used $\rho(q)$ to denote the value of $\rho$ for consumer with size $q$. ■

**Proof of Lemma 2.** Let the price, denoted $P$, equal the minimum quote. We first derive the distribution of prices using the quote distribution characterized above. If the buyer only gets one quote, the density of prices is the same as the quote density. If the buyer gets two quotes, he will accept the minimum of the two. The density of prices is given by $2(1 - F(p; q; \rho)) f(p; q; \rho)$. Given that buyers only get one quote with probability $\rho$, one finds the density of prices as,

$$f_P(p; q) = \rho f(p; q) + (1 - \rho) 2(1 - F(p; q)) f(p; q)$$

$$= \rho^2 (2v - p) v.$$ 

The cumulative distribution function of prices is thus given by

$$F_P(p; q) = \frac{(p - \rho v)(\rho v + (1 - \rho)p)}{p^2(1 - \rho)}.$$ 

18
We first show that the price distribution $F_P(p; q)$ is decreasing in $\rho$,

$$\frac{\partial F_P(p; q)}{\partial \rho} = -v \rho (2 - \rho) \frac{v - p}{p^2 (1 - \rho)^2} < 0.$$ 

Expected prices are given by

$$E(P; q) = \int_p^\varnothing p dF_P(p; q) = v \frac{\rho}{1 - \rho} (2 (1 - \rho) + \rho \ln \rho).$$

The first derivative with respect to $\rho$ is positive,

$$\frac{\partial E(P; q)}{\partial \rho} = v (2 - \rho) \frac{1 - \rho + \rho \ln \rho}{(1 - \rho)^2} > 0.$$ 

The variance of prices is given by

$$Var(P; q) = \int_p^\varnothing p^2 dF_P(p; q) - Var^2(P; q) = -v^2 \rho^2 \left(2 \ln \rho + (1 - \rho) + \frac{1}{1 - \rho} (2 (1 - \rho) + \rho \ln \rho)^2\right).$$

It can be shown that $Var(P; q)$ is monotonically increasing in $\rho \in (0, 0.306236)$, and monotonically decreasing in $\rho \in (0.306236, 1)$. Hence, it achieves a maximum at 0.306236, as depicted in Figure 2. Furthermore, $Var(P; q) = 0$ for $\rho \to 0$ and $\rho \to 1$.  

Figure 2: Variance of quotes (thin) and variance of prices (thick)
Proof of Proposition 2. A buyer who asks for either one or two quotes derives utility

\[ u_1(q,c) = \left( v - \int_p^\varpi p dF(p;q) \right) q - c \]

\[ u_2(q,c) = \left( v - \int_p^\varpi 2p (1 - F(p;q)) dF(p;q) \right) q - 2c \]

with \( F(p;q) \) and \([\underline{p}, \overline{p}]\) as characterized in Proposition 1. The functions \( u_1(q,c) \) and \( u_2(q,c) \) are decreasing in \( c \) with constant slope \(-1\) and \(-2\), respectively, and \( u_2(q,0) > u_1(q,0) > 0 \). Furthermore, they are both proportional to \( q \). It follows that there exist \( c_0 > c_1 > 0 \) such that (i) \( 0 > u_1(q,c) > u_2(q,c) \) for \( c > c_0 \); (ii) \( u_1(q,c) \geq \max \{ u_2(q,c), 0 \} \) for \( c \in (c_1, c_0] \) and (iii) \( u_2(q,c) \geq u_1(q,c) > 0 \) for \( c \leq c_1 \). More specifically, \( c_0 \) and \( c_1 \) are implicitly defined by \( u_1(q,c_0) = 0 \) and \( u_1(q,c_1) = u_2(q,c_1) \). Explicitly, using the equilibrium price distribution characterized in Proposition 1,

\[ c_0 = q \left( v - \int_p^\varpi p dF(p;q) \right) = vq \left( 1 - \frac{\rho}{1 - \rho} \ln \frac{1}{\rho} \right) > 0 \]

and

\[ c_1 = q \int_p^\varpi (2F(p;q) - 1) p dF(p;q) = -vq \frac{\rho}{1 - \rho} \left( \frac{1 + \rho}{1 - \rho} \ln \rho + 2 \right) > 0. \]

Note that

\[ c_0 - c_1 = vq \left( 1 - \frac{\rho}{1 - \rho} \ln \frac{1}{\rho} \right) + vq \frac{\rho}{(1 - \rho) \ln \rho + 2} \geq 0 \]

with equality for \( \rho \to 1 \).

The first and second derivatives of \( c_0 \) are given by,

\[ \frac{\partial c_0}{\partial \rho} = -\frac{vq}{(1 - \rho)^2} \left( \rho + \ln \frac{1}{\rho} - 1 \right) < 0 \]

\[ \frac{\partial^2 c_0}{\partial \rho^2} = \frac{vq}{\rho (1 - \rho)^3} \left( \rho^2 + 2\rho \ln \frac{1}{\rho} - 1 \right) > 0 \]

Note that \( c_0 \) is decreasing in \( \rho \), with \( c_0 = v \) for \( \rho \to 0 \) and \( c_0 = 0 \) for \( \rho \to 1 \). Furthermore,

\[ \lim_{\rho \to 1} \frac{\partial c_0}{\partial \rho} = -\frac{vq}{2}. \]
The first and second derivatives of \( c_1 \) are given by,
\[
\frac{\partial c_1}{\partial \rho} = \frac{vq}{(1 - \rho)^3} ((\rho + 3) (1 - \rho) + \ln \rho (1 + 3\rho))
\]
\[
\frac{\partial^2 c_1}{\partial \rho^2} = -\frac{vq(1 - \rho)(10\rho + \rho^2 + 1) + 6\rho \ln \rho (1 - \rho)}{\rho (1 - \rho)^4} < 0.
\]

The function \( c_1 \) is non-monotonic in \( \rho \). A sufficient condition for \( c_1 \) to be decreasing in \( \rho \) is \( \rho \geq 0.223 \).

Note that \( c_1 \) is concave in \( \rho \), with \( \lim_{\rho \to 0} c_1 = \lim_{\rho \to 1} c_1 = 0 \), and
\[
\lim_{\rho \to 1} \frac{\partial c_1}{\partial \rho} = -\frac{vq}{6}.
\]

**Proof of Theorem 1.** We need to show that there exists a solution to (1) in \((0, 1)\). As shown in the main text, equation (1) can be expressed as the piece-wise function
\[
\rho = \begin{cases} 
1 - \frac{G(c_1(\rho))}{G(c_0(\rho))} & \text{if } \rho \leq \bar{\rho} \\
1 - G(c_1(\rho)) & \text{if } \rho \geq \bar{\rho}
\end{cases}
\]
where \( \bar{\rho} \) is implicitly defined by \( c_0(\bar{\rho}) = \bar{c} \).\(^{31}\) Since \( c_0(\rho) \) and \( c_1(\rho) \) are continuous in \( \rho \), the two pieces are continuous in \( \rho \). The function is also continuous at \( \bar{\rho} \) given that \( G(c_0(\bar{\rho})) = 1 \).

To show that condition (1) has a solution, note that
\[
\lim_{\rho \to 0} (1 - G(c_1(\rho))) = 1
\]
since \( \lim_{\rho \to 0} c_1(\rho) \). Furthermore,
\[
\lim_{\rho \to 1} \left( 1 - \frac{G(c_1(\rho))}{G(c_0(\rho))} \right) = \frac{2}{3}.
\]

To see this, note that \( \lim_{\rho \to 1} c_0(\rho) = \lim_{\rho \to 1} c_1(\rho) = 0 \), implying that both \( G(c_1(\rho)) \) and \( G(c_0(\rho)) \) take the value 0 for \( \rho \to 1 \), so \( \lim_{\rho \to 1} \frac{G(c_1(\rho))}{G(c_0(\rho))} \) is undefined. Applying l'Hôpital,
\[
\lim_{\rho \to 1} \left( 1 - \frac{G(c_1(\rho))}{G(c_0(\rho))} \right) = 1 - \lim_{\rho \to 1} \frac{G'(c_1(\rho)) \frac{\partial c_1}{\partial \rho}}{G'(c_0(\rho)) \frac{\partial c_0}{\partial \rho}} = 1 - \frac{G'(0) vq/6}{G'(0) vq/2} = \frac{2}{3}.
\]

Since the function is continuous, it takes the value 1 for \( \rho \to 0 \) and a lower value for \( \rho \to 1 \), it follows that the schedule must cross the 45 degree line at some \( \rho^* \in (0, 1) \). Hence, there exists an

\[\text{Note that } c_0(1) = v \text{ so that } \bar{\rho}(q) = 0 \text{ for } q \leq \bar{c}/v \text{ and } \bar{\rho}(q) > 0 \text{ for } q > \bar{c}/v.\]
interior solution to (1).

One can also show that, under certain conditions, the solution is unique, i.e., there is a single fixed point to Equation (1). First, we show that each of the two segments can only cross the 45-degree line once. Having each of the lines cross the 45-degree line once is sufficient to show that the equilibrium is unique. The reason is that Equation (1) is defined as the continuous function of the two segments, and we will show that in equilibrium, the two functions cross the 45-degree line from above and do not cross it again.

It is useful to think about the two regions as being the same function $1 - \frac{G(c_1)}{G(c_0)}$, just that in some regions $c_0 = \bar{c}$ and therefore $G(c_0)$ is constant and equal to 1.

The general derivative for the function is,

$$\frac{\partial}{\partial \rho} \left( 1 - \frac{G(c_1)}{G(c_0)} \right) = -\frac{G'(c_1) G(c_0) \frac{\partial c_1}{\partial \rho} - G'(c_0) G(c_1) \frac{\partial c_0}{\partial \rho}}{(G(c_0))^2}. $$

**Case 1.** In the corner solution, $c_0$ does not change (it is at its max). The relevant numerator becomes,

$$-G'(c_1) \frac{\partial c_1}{\partial \rho}. $$

It implies that $1 - G(c_1)$ is initially decreasing (for $\rho < 0.223$, as shown in Figure 1), and increasing afterwards. Convexity ensures it only crosses the 45-degree line once, which happens as long as the second order derivative $-G'' \left( \frac{\partial c_1}{\partial \rho} \right)^2 - G' \frac{\partial^2 c_1}{\partial \rho^2} > 0$, which is true for $G'' < 0$, given concavity of $c_1$.

**Case 2.** In the interior range for $c_0$, relevant numerator becomes,

$$-G'(c_1) G(c_0) \frac{\partial c_1}{\partial \rho} - G'(c_0) G(c_1) \frac{\partial c_0}{\partial \rho}. $$

It will be negative as long as,

$$\frac{G'(c_1) \frac{\partial c_1}{\partial \rho}}{G(c_1)} > \frac{G'(c_0) \frac{\partial c_0}{\partial \rho}}{G(c_0)}. $$

A sufficient condition on this is MLRP of $G$ (e.g. this is satisfied by the uniform). The reason is that

$$\frac{\partial c_1}{\partial \rho} > \frac{\partial c_0}{\partial \rho},$$

so it is sufficient that,

$$\frac{G'(c_1)}{G(c_1)} > \frac{G'(c_0)}{G(c_0)}. $$

**Proof of Proposition 3.** We first show that the schedule $(1 - G(c_1(\rho)))$ is strictly convex in $\rho$. Taking derivatives,

$$\frac{\partial}{\partial \rho} (1 - G(c_1(\rho))) = -G'(c_1(\rho)) \frac{\partial c_1}{\partial \rho}. $$
\[
\frac{\partial^2 (1 - G(c_1(\rho)))}{\partial \rho^2} = -G''(c_1(\rho)) \left( \frac{\partial c_1}{\partial \rho} \right)^2 - G'(c_1(\rho)) \frac{\partial^2 c_1}{\partial \rho^2} > 0.
\]

The first term of the second-order derivative is positive since \(G'' \leq 0\); the second term is also positive because \(\frac{\partial^2 c_1}{\partial \rho^2} < 0\).

Furthermore, from \(\lim_{\rho \to 0} c_1(\rho) = \lim_{\rho \to 1} c_1(\rho) = 0\), it follows that \(\lim_{\rho \to 0} (1 - G(c_1(\rho))) = \lim_{\rho \to 1} (1 - G(c_1(\rho))) = 0\). Hence, whenever the slope of \((1 - G(c_1(\rho)))\) at \(\rho = 1\) is below 1, \((1 - G(c_1(\rho)))\) does not cross the 45-degree line. In contrast, when the slope at \(\rho = 1\) is above 1, it must cross the 45-degree line at some \(\rho \in (0, 1)\).

Let \(\overline{q}\) be such that the 45-degree line is tangent to \((1 - G(c_1(\rho)))\) at \(\rho = 1\),

\[
\lim_{\rho \to 1} \frac{\partial (1 - G(c_1(\rho)))}{\partial \rho} = \lim_{\rho \to 1} \left( -G'(c_1(\rho)) \frac{\partial c_1}{\partial \rho} \right) = G'(0) \frac{\nu}{6} = 1 \implies \overline{q} = \frac{6}{\nu G'(0)}.
\]

Furthermore, as \(q\) goes up, the schedule \((1 - G(c_1))\) shifts down,

\[
\frac{\partial (1 - G(c_1))}{\partial q} = -G'(c_1) \frac{c_1}{q} < 0.
\]

It thus follows that when \(q < \overline{q}\), the schedule \((1 - G(c_1(\rho)))\) cannot cross the 45-degree line.

Since a solution exists (Theorem 1), then it must be as a solution to \(\left(1 - \frac{G(c_1(\rho))}{c_0(\rho)}\right) = \rho^*\). In contrast, for \(q > \overline{q}\), the function \((1 - G(c_1(\rho)))\) crosses the 45-degree line at some \(\rho^*\) which solves \((1 - G(c_1(\rho^*))) = \rho^*\). To conclude the proof, we thus need to show that for \(q > \overline{q}\), \(\overline{\rho} \geq \rho^*\). To see this, note that \(\overline{\rho}\) is implicitly defined by \(c_0(\overline{\rho}) = \overline{c}\). Since \(c_0\) is increasing in \(q\) and decreasing in \(\rho\) and \(\overline{c}\) is a constant, it follows that \(\overline{\rho}\) must be increasing in \(q\). For \(q = \overline{q}\), \(\overline{\rho} = \rho^*\) [PENDING: need to show that at \(q = \overline{q}\), \(\overline{\rho} = \rho^*\)]. It follows that for \(q > \overline{q}\), we must have \(\overline{\rho} > \rho^*\). This concludes the proof. ■

**Proof of Proposition 4.** By Lemma 2, expected prices are increasing in \(\rho\). Hence, to assess the comparative statics of expected prices with respect to \(q\), it suffices to assess the comparative statics of the solution to equation (1) with respect to \(q\).

(i) For expected prices to be decreasing in \(q\), the solution to \(\rho = 1 - G(c_1)\) must be decreasing in \(q\). Indeed, as \(q\) goes up, the schedule \((1 - G(c_1))\) shifts down, as shown before. Hence, \(1 - G(c_1)\) crosses the 45-degree line at a smaller value of \(\rho\), which in turn leads to lower expected prices.

(ii) For expected prices to be decreasing in \(q\), the solution to \(\rho = 1 - \frac{G(c_1)}{G(c_0)}\) must be decreasing in \(q\); equivalently, the ratio \(\frac{G(c_1)}{G(c_0)}\) must be increasing in \(q\). This is the case if and only if

\[
\frac{\partial \left( \frac{G(c_1)}{G(c_0)} \right)}{\partial q} = \frac{c_1 G(c_0) G'(c_1) - c_0 G(c_1) G'(c_0)}{q (G(c_0))^2} > 0.
\]
Since the denominator is positive, we focus attention on the numerator, which can be re-written as

\[ G(c_0) G(c_1) \left[ c_1 \frac{G'(c_1)}{G(c_1)} - c_0 \frac{G'(c_0)}{G(c_0)} \right], \]

or using the expression for \( \varepsilon(c) \),

\[ G(c_0) G(c_1) [\varepsilon(c_1) - \varepsilon(c_0)]. \]

It follows that a necessary and sufficient condition for \( 1 - \frac{G(c_1)}{G(c_0)} \) to be decreasing in \( q \) is that \( \varepsilon(c_1) \geq \varepsilon(c_0) \), which is satisfied whenever the elasticity \( \varepsilon(c) \) is decreasing in \( c \), as claimed. Hence, as \( q \) goes up, \( 1 - \frac{G(c_1)}{G(c_0)} \) crosses the 45-degree line at a smaller value of \( \rho \), which in turn leads lower expected prices.  

**Proof of Proposition 5.**

(i) For expected prices to be decreasing in \( \lambda \), the solution to \( \rho = 1 - G(c_1; \lambda) \) must be decreasing in \( \lambda \). Indeed, as \( \lambda \) goes up, the schedule \( (1 - G(c_1; \lambda)) \) shifts down, so that it crosses the 45-degree line at a lower value of \( \rho \), which in turn leads to lower expected prices.

(ii) For expected prices to be decreasing in \( \lambda \), the solution to \( \rho = 1 - \frac{G(c_1; \lambda)}{G(c_0; \lambda)} \) must be decreasing in \( \lambda \). In turn, the ratio \( \frac{G(c_1; \lambda)}{G(c_0; \lambda)} \) is increasing in \( \lambda \) iff:

\[
\frac{\partial}{\partial \lambda} \left( \frac{G(c_1; \lambda)}{G(c_0; \lambda)} \right) = G(c_0; \lambda) \frac{\partial G(c_1; \lambda)}{\partial \lambda} - G(c_1; \lambda) \frac{\partial G(c_0; \lambda)}{\partial \lambda} > 0.
\]

Since the denominator is positive, we focus on the numerator, which can be re-written as:

\[
\frac{1}{G(c_0; \lambda) G(c_1; \lambda)} \left[ \frac{\partial G(c_1; \lambda)}{\partial \lambda} \frac{\lambda}{G(c_1; \lambda)} - \frac{\partial G(c_0; \lambda)}{\partial \lambda} \frac{\lambda}{G(c_0; \lambda)} \right] > 0.
\]

Let us define the elasticity with respect to \( \lambda \) for given \( c \) as

\[ \varepsilon(\lambda) \equiv \frac{\partial G(c; \lambda)}{\partial \lambda} \frac{\lambda}{G(c; \lambda)}. \]

Hence, focusing on the term in brackets, the ratio \( \frac{G(c_1; \lambda)}{G(c_0; \lambda)} \) is increasing in \( \lambda \) if the elasticity at \( c_1 \) is higher than at \( c_0 \). Since \( c_1 < c_0 \), a sufficient condition is that the elasticity with respect to \( \lambda \) is decreasing in \( c \). For constant \( \varepsilon(\lambda) \), the equilibrium condition does not change with \( \lambda \).  

**Proof of Lemma 3.** Using Proposition 3, \( \overline{\rho}(\lambda) = 6/vG'(0; \lambda) \). To prove the Lemma, simply note that \( \overline{\rho}(\lambda) \) is decreasing in \( \lambda \) given that \( \partial G(c; \lambda) / \partial \lambda > 0 \).  

**Proof of Proposition 7.** Similar arguments are those in Proposition 1 allow us to conclude that there does not exist an equilibrium in pre-strategies.

Let seller \( i \)'s profits from quoting a price \( p \) when rivals are choosing quotes according to \( F(p; q) \)
be given by:

$$\pi (p) = p \int q [\rho (q) + (1 - \rho (q)) (1 - F (p))] dH (q)$$

Let $x = \int q \rho (q) dH (q)$ and $y = \int q dH (q)$ with $x < y$, then we can write is as

$$\pi (p) = p (x + (y - x) (1 - F (p)))$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi (p) = \pi \text{ for all } p \in [p, \bar{p}] .$$

Furthermore, as $\pi (\bar{p}) = \bar{p}y$ is increasing in $\bar{p}$, it follows that $\bar{p} = v$. Hence, $\pi = vx$. From $\pi (\bar{p}) = \bar{p}y = \pi = vx$ it also follows that $p = \frac{x}{y} v$. To obtain the equilibrium quote distribution, from $\pi (p) = vx$, it follows that

$$F (p) = 1 - \frac{x}{y - x} \frac{v - p}{p}$$

Renormalize it by defining $\hat{\rho} = \frac{x}{y} = \frac{\int q \rho (q) dH (q)}{\int q dH (q)} < 1$ (i.e., $\hat{\rho}$ is the weighted average $\rho$) so that

$$F (p) = 1 - \frac{\hat{\rho}}{1 - \hat{\rho}} \frac{v - p}{p} .$$

Sellers’ expected profits are thus

$$\pi = v \int q \rho (q) dH (q) .$$

\vspace{1cm}