Industry Costs and Research Aggregation in Dynamic Competition

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Abstract

We study information acquisition and subsequent price competition in an environment where the cost of each firm is initially unknown and composed of two components, private costs specific to the firm and costs common to all firms in the industry. In this setting, firms choose high initial prices to soften future competition. Moreover, this pricing distortion is exacerbated when firms only possess private information about firm specific costs. This implies that sharing information about industry relevant costs, such as aggregating cost information through a trade association, will lead to higher prices. Additionally, when firms share information about common costs they have less incentive to acquire information about firm specific costs which can lead to lower expected profits for firms.

1 Introduction

Firms in durable goods markets are frequently imperfectly aware of their own costs of production prior to market entry.[1] While information improves over time during the production process, prices are initially set based off firms' forecasts about the cost of production. Firms have an incentive to soften future competition with rival firms by choosing a high price in the Bertrand setting [Mailath, 1989], or a large output in the Cournot setting [Bonatti et al., 2017]. Moreover, Jeitschko et al. [2017] shows this strategic effect can reduce firms' incentive to make precise forecasts prior to beginning the production process, leading to

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[1] That costs are imperfectly known can arise from, for example, learning by doing on the part of the firm's suppliers, cite here, or from a volatile external market for inputs.

reduced welfare in the market. While past work has focused on these strategic incentives for firm-specific cost information, firms often have information about industry-wide costs, e.g. forecasted prices of common inputs, in addition to private production costs. Additionally, it has been shown that the incentive to share cost information depends crucially on which type of information it is.[2] Despite this, the strategic effects of acquiring and sharing information on the cost of production have not been studied in a setting where firms can distinguish information on their private and common cost components.

In this paper, we study information acquisition and subsequent price competition in an environment where the cost of each firm is initially unknown and composed of two components, private costs specific to the firm and costs common to all firms in the industry. We show that the pricing distortions which arise from dynamic pricing competition are less severe in this setting than when firms only posses information about a private cost parameter. Moreover, the problem of initial underinvestment in information that stems from this distortion is alleviated, leading firms to acquire a more efficient level of information prior to production. The reverse implication of this is that when firms in the industry share common cost information via a trade association, pricing distortions are exacerbated and the incentive to acquire information prior to beginning production falls.

We consider firm competition that takes place over three stages. First, prior to beginning the production process and setting prices, firms can expend resources to acquire information about each cost component. The firms then engage in price competition over two periods. Firms initially choose a price based on the information they receive in the acquisition stage. After the first period of competition, each firm obtains full information of its own private costs and the costs that are common to the industry, and uses observed first-period prices to update its beliefs about the private cost component of its competitor. With this additional information, firms again choose prices in the second stage of price competition.

We first show that there is a unique symmetric equilibrium in linear strategies of the two period pricing game, given any level of information precision acquired by the firms. The effects of information on price depend on whether goods are substitutes or complements. When goods are substitutes information about common costs is weighted more heavily when determining first period prices; when goods are complements information about private costs is weighted more heavily when determining first period prices. The main force behind this difference is firms' responses to direct information about their competitors' cost through the common cost component. When firms sell substitutable goods, then prices are strategic complements, and a high cost signal leads to higher expected demand. When firms are selling

[2] See Raith [1996], Sankar [1995] and Ackert et al. [2000]. For a survey of the information sharing literature see Vives [2001]

complementary goods, prices are strategic substitutes so a high common cost signal leads to reduced expected demand.

A firm's first period price reveals information about its cost structure, which its opponent can use in second period competition. This directly affects first period price selection. Specifically, when competitors observe the firm setting a high price they believe that the firm has a high expected cost of production. This causes the competing firms to set a higher price in the second period, softening the price competition in this second period for the firm that set a high price in the first period. Therefore, each firm has an incentive to over represent their costs by choosing higher initial prices.

In the setting of two cost components, this high expectation of cost could stem from the firm receiving a high signal about industry-common costs, firm-specific costs, or both. Because only firm-specific information is uncertain in the second round, firms use the first period price to update their beliefs about these costs specifically. Therefore the initial uncertainty of industry relevant costs adds noise to the relationship between the first period price and the private information about firm-specific costs. When firms share industry-common cost information via a trade association prior to setting initial prices, private information about industry relevant information disappears and first period prices become a clearer signal of the firm's private information about its idiosyncratic costs. This intensifies the incentive to over-represent costs and results in higher expected initial prices.

Given the pricing equilibrium and resulting profits of the firms, we analyze the incentive to acquire information prior to the two-stage competition. Firms can improve the precision of their information by hiring a private consultant (for example) to give information on both industry-common and firm-specific costs. Once firms have this information, we again consider two settings: either the information stays private to each firm prior to the two stage competition, or firms share industry-relevant information via a trade association. When firms share information they have less incentive to acquire information about firm-specific costs; additional precision of information leads to less expected profits in both periods of price competition. In the initial stage of price competition pricing distortions are exacerbated, reducing the variance of price choice relative to the available information and therefore making this information less valuable to the firms.[3] Moreover, initial prices fully reveal the private information of the firm prior to the second period of competition. When industry costs are not shared, initially acquired information about idiosyncratic costs persists and remains valuable to firms in this second period.[4]

This paper generalizes the model in Jeitschko et al. [2017], allowing for initial uncertainty

[3]This effect is thoroughly discussed in Jeitschko et al. [2017].

[4]See Gal-Or [1986].

and acquisition of information in both private firm-specific costs and common industry-wide costs. One major technical difference to this model is the increased dimensionality of the strategy space. Because private information in the initial stage of competition is of a higher dimension than the action space, a fully separating strategy cannot exist and information is not fully revealed to competing firms prior to the second round of competition. Therefore private information persists into the second round of competition without the presence of hidden actions as in the signal jamming literature (e.g., Mirman et al. [1993]) or adding additional exogenous noise in later rounds (e.g., Mester [1992]).

The ability of firms to distort private information ties this paper to the literature on information manipulation in oligopolistic competition. In Mirman et al. [1994] firms have private information about a common demand parameter in a dynamic duopoly setting. It is shown that when information has a negative net value, as is the case in both our setting and in Jeitschko et al. [2017], firms decrease the informativeness of their strategic choice. In supply schedule competition, Vives [2011] shows that an increase in correlation of costs leads to larger strategic distortions, while a more precise public signal on the common value component can alleviate these distortions. This is consistent with our result that the public release of a common cost signal increases the incentive to distort information on the private cost component. Additionally, Bernhardt and Taub [2015] examines the impact of private information in common-valued and private-valued coefficients on both firm costs and market demand showing that private value information distorts supply choice more heavily than common valued information.

Lastly, this paper adds to the growing literature which evaluates the welfare impacts of information acquired endogenously. It is shown in Colombo et al. [2014] that the optimal precision of private information will decrease in the precision of publicly available common-valued information in a general quadratic-Gaussian setting. In the context of Cournot competition, Myatt and Wallace [2015a] show that firms will inefficiently acquire too much information about uncertain demand and use inefficiently too little of the information when making quantity choices. Additional work analyzes efficient use of acquired information and its impact on social welfare, e.g. Angeletos and Pavan [2007] and Myatt and Wallace [2015b].

2 Model

Two firms, i and j, compete for market share over two periods $t = 1, 2$. Demand is linear in prices, symmetric across firms, and time-independent. Firm i's demand is given by

$$q_{i,t} = a - bp_{i,t} + ep_{j,t}.$$ [5]

We assume that demand is weakly more sensitive to a firm's own price than to its opponent's, so that $|e| \leq b$. Each firm faces a constant marginal cost c_i that is same in each period, so profits are

$$\pi_{i,t} = (p_{i,t} - c_i)\, q_{i,t}.$$

Firms are initially uncertain about their marginal costs of production, but know that costs are comprised of an idiosyncratic component θ_i and a common component ρ; their constant marginal cost is the sum of the two components, $c_i = \rho + \theta_i$. We assume that cost components are joint-normally distibuted with zero covariance, so that

$$\begin{pmatrix} \theta_i \\ \theta_j \\ \rho \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \\ \mu_\rho \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\rho^2 \end{pmatrix} \right).$$

Throughout we will denote the precision of the random variable x by $\tau_x = 1/\sigma_x^2$.

Play proceeds in two stages. In the first stage, each firm receives two noisy signals, $s_{i,\theta}$ and $s_{i,\rho}$, of the values of their idiosyncratic and common costs, respectively. These signals are normally distributed with uncorrelated error terms, and the error terms are uncorrelated between firms. We model these signals as $s_{i,x} = x + \varepsilon_{i,x}$, where $\varepsilon_{i,x}$ is normally distributed with variance $\sigma^2_{\varepsilon,x,i}$.[6] Upon the realization of their private signals, firms simultaneously select prices $p_{i,1}$ and obtain stage profits $\pi_{i,1}$.

After first-stage profits are obtained, firms become perfectly informed of both the common and their (individual) idiosyncratic cost components. Each also witnesses its opponent's first-stage price, but remains unaware of its opponent's idiosyncratic cost component.[7] Firms then compete again by simultaneously selecting prices and obtain stage profits $\pi_{i,2}$.

The game ends after the second stage, and ex post utility is the (undiscounted) sum of stage profits,

$$u_i\,(p_i, p_j) = \pi_{i,1}\,(p_{i,1}, p_{j,1}) + \pi_{i,t}\,(p_{i,2}, p_{j,2})\,.$$

We restrict attention to *subgame perfect equilibria in linear strategies.*

Analysis proceeds in three parts. We first determine properties of equilibrium in this base model. Then, we allow firms to share information about their common cost component prior to the two-stage competition. Lastly, we allow firms to acquire more precise information

[5]Unless otherwise specified, our equations and inequalities should be taken to be symmetric for agent j.

[6]For the majority of our results we will assume symmetry, so that $\sigma^2_{\varepsilon,x,i} = \sigma^2_{\varepsilon,x,j}$. Allowing for heterogeneity in the variance of the error term is essential when we discuss information acquisition.

[7]Since demand is a deterministic function of firm prices, the assumption that firms witness each others' prices is sufficient to imply that they are perfectly informed of their own private cost c_i; alternatively, if they witness their own sales volume they will be perfectly aware of their opponent's price. That they obtain perfect knowledge of each of the components of $c_i = \rho + \theta_i$ is an additional assumption.

about their costs, and compare the amount of information acquired in the setting where firms share common cost information to the setting where they do not.

3 Equilibrium

We compute the pricing equilibrium in the two stage model by backwards induction. In a subgame-perfect equilibrium second-period prices are best responses to available information. However, even in an equilibrium where first period prices are strictly monotone in each signal, it is impossible for private information to be fully-revealed as in a standard separating equilibrium as private information is two-dimensional while actions are one-dimensional and monotone in information. Residual uncertainty in the second stage is an important feature in our model, affecting firms' first-period pricing through their ability to distort publicly-available information about their costs.

3.1 Second period pricing

In the second period, each firm knows its own marginal costs precisely, but knows only the distribution over its opponent's costs. Letting $F^j(\cdot; p_{i,1}, p_{j,1}, \rho) \equiv F^j$ be the distribution of firm j's second period price conditional on firm i's available information,[8] the profit maximization problem is

$$\max_p \int (p - c_i)(a - bp + ex)\, dF^j(x).$$

Lemma 1. *Firm i's optimal second period price is*

$$p^\star_{i,2} = \frac{1}{2b}\left(a + bc_i + e\mathbb{E}\left[p^\star_{j,2}\middle|\, \rho, p_{i,1}, p_{j,1}\right]\right).$$

Firm i's maximum second period expected profit is

$$\pi^\star_{i,2} = \frac{1}{4b}\left(a - bc_i + e\mathbb{E}\left[p^\star_{j,2}\middle|\, \rho, p_{i,1}, p_{j,1}\right]\right)^2.$$

Thus firm i's second period price is an affine function of the demand intercept, its (known) cost $c_i = \rho + \theta_i$, and its expectation over firm j's second period price. Profits then have a standard quadratic form.

[8] Firm i also knows θ_i, $s_{i,\theta}$, and $s_{i,\rho}$, but these offer no payoff-relevant information in the second stage (beyond θ_i, ρ, and $p_{i,1}$) and may be ignored.

Lemma 2. *In any equilibrium, expected second period prices of a firm given publically available information are*

$$\mathbb{E}\left[p^{\star}_{j,2} \middle| \rho, p_{i,1}, p_{j,1} \right] = \frac{1}{4b^2 - e^2} \left((2b + e)\, a + 2b^2 \mathbb{E}\left[c_j \middle| \rho, p_{j,1} \right] + be \mathbb{E}\left[c_i \middle| \rho, p_{i,1} \right] \right),$$

which result in the following expected second period profits:

$$\pi^{\star}_{i,2} = \frac{1}{4b} \left(\frac{1}{4b^2 - e^2} \right)^2 \left(\left(4b^2 + 2be\right) a - 4b^3 c_i + \left(\mathbb{E}\left[c_i \middle| \rho, p_{i,1} \right] - c_i \right) be^2 + 2b^2 e \mathbb{E}\left[c_j \middle| \rho, p_{j,1} \right] \right)^2.$$

Lemma 2 connects firms i's expected second period profits to its first period price. These profits increase in $E[c_i|\rho, p_{i,1}]$, the expectation of firm i's cost given information available to firm j in the second period. Therefore firm i has an incentive to over-represent its cost, leading firm j to increase its second period price, softening competition for firm i.[9]

3.2 First period pricing

First period prices are set to optimize the sum of profits over two periods. Although first period prices have no direct effect on second period profits, firm i's price affects firm j's beliefs regarding firm i's costs. This is shown directly in Lemma 2, where $p_{i,1}$ enters only in $\mathbb{E}[c_i|\rho, p_{i,1}]$.

Firm i's first period profit maximization problem is

$$\max_p \mathbb{E}\left[\pi_{i,1} \middle| s_{i,\rho}, s_{i,\theta} \right] + \mathbb{E}\left[\pi^{\star}_{i,2} \middle| s_{i,\rho}, s_{i,\theta} \right] = \max_p \mathbb{E}\left[(a - bp + e\hat{p}_{j,1})\,(p - c_i) + \pi^{\star}_{i,2} \middle| s_{i,\rho}, s_{i,\theta} \right].$$

A marginal increase in first period price affects first period profits in a standard way, and has an additional effect on second period profits by manipulation of the opposing firm's second period beliefs which changes second period price choices. This gives Lemma 3.

Lemma 3. *Optimal first period prices are given by*

$$\begin{aligned} p^{\star}_{i,1} = {} & \left(\frac{1}{2b} \right) \mathbb{E}\left[bc_i + a + e\hat{p}_{j,1} \middle| s_{i,\rho}, s_{i,\theta} \right] \\ & + e \left(\frac{1}{2b} \right)^2 \mathbb{E}\left[\left(a - bc_i + e\mathbb{E}\left[p^{\star}_{j,2} \middle| \rho, p^{\star}_{i,1}, \hat{p}_{j,1} \right] \right) \frac{\partial}{\partial p_{i,1}} \mathbb{E}\left[p^{\star}_{j,2} \middle| \rho, p^{\star}_{i,1}, \hat{p}_{j,1} \right] \middle| s_{i,\rho}, s_{i,\theta} \right]. \end{aligned}$$

We constrain attention to equilibria in pricing strategies that are linear in the expected

[9] This describes the reaction of firm j when the firms are selling substitutes, $e > 0$. When $e < 0$, a higher value of $E[c_i|\rho, p_{i,1}]$ leads to a lower $p^{*}_{j,2}$ which still increases $\pi^{*}_{i,2}$. When $e = 0$, the price and profit equations reduce to the standard monopoly model.

value of each cost component.[10] A linear first period price can be expressed as

$$p_{i,1} = p_{i,0} + \mathbb{E}\left[\theta_i \middle| s_{i,\theta}\right] p_{i,\theta} + \mathbb{E}\left[\rho \middle| s_{i,\rho}\right] p_{i,\rho}.$$

Under linear strategies, each firm's first period price choice is a normally distributed random variable from the perspective of the other firm. Therefore, $(c_i, \rho, p_{i,1})$ are distributed joint-normally, which implies that $\mathbb{E}[c_i|\rho, p_{i,1}]$ is linear in $p_{i,1}$. Moreover, the effect of an increase in firm i's first period price on firm j's second period beliefs, and hence second period price, is constant and independent of the level of price. Conditioning beliefs on this relationship gives Lemma 4.

Lemma 4. *The marginal effect of firm i's first period price on firm j's expected second period price is*

$$\frac{\partial}{\partial p_{i,1}} \mathbb{E}\left[p^{\star}_{j,2} \middle| \rho, p_{i,1}\right] = \frac{be}{4b^2 - e^2}\kappa_i,$$

$$\textit{where } \kappa_i \equiv \frac{\partial}{\partial p_{i,1}} \mathbb{E}\left[c_i \middle| \rho, p_{i,1}\right] = \frac{\sigma^2_\theta \bar{\tau}_{s,\theta,i} p_{i,\theta}}{\sigma^2_{s,\rho}\bar{\tau}^2_{s,\rho,i}p^2_{i,\rho} + \left(\sigma^2_\theta + \sigma^2_{s,\theta,i}\right)\bar{\tau}^2_{s,\theta,i}p^2_{i,\theta}} \textit{ and } \bar{\tau}_{s,x,i} = \frac{\tau_{s,x,i}}{\tau_x + \tau_{s,x,i}}.$$

The term κ_i captures the relative informativeness of firm i's first period price regarding the its idiosyncratic cost component θ_i, the remaining source of asymmetric information in the second period when ρ is commonly known. Despite observing ρ, firms do not observe each other's first period signal on the common cost component, $s_{i,\rho}$. Because the first period price depends on the realization of $s_{i,\rho}$, it can be thought of a noisy signal of $s_{i,\theta}$, Therefore the informativeness of the price in determining θ_i depends not only on the variance of the price relative to $s_{i,\theta}$ but also relative to $s_{i,\rho}$.[11]

For $x \in \{\rho, \theta\}$, $\bar{\tau}_{s,x,i}$ is the relative contribution of the normally distributed noise in firm i's signal around the true parameter x, to the precision of the signal $s_{i,x}$. When signals are very noisy, $\bar{\tau}_{s,x,i}$ will be close to zero; when signals give a more precise prediction of the true cost parameter, $\bar{\tau}_{s,x,i}$ will be close to one. When signals are more precise, they have a larger role in the formation of expectations over the cost parameters.

The term $\bar{\tau}_{s,x,i}p_{i,x}$ is the derivative of first period price with respect to $s_{i,x}$, and affects the informativeness of the first period price about the firms cost. Therefore the choice of strategy in the first period for a given level of information precision will directly impact the

[10] This forces each firm to commit to using information about each cost component at a fixed level for all possible signals, $(s_{i,\theta}, s_{i,\rho})$, it may receive. As we show, these strategies are best responses to the opponent's linear pricing rule, even allowing for nonlinear pricing rules, when the firm has received its private signals. It is possible that there exist equilibria in nonlinear pricing rules.

[11] Note that σ^2_ρ does not appear in the denominator of κ_i since ρ is commonly observed.

value of κ_i. Specifically, as either $p_{i,\theta}$ or $p_{i,\rho}$ increases, κ_i decreases. If a firm increases $p_{i,x}$ while precions remains constant, it is increasing the variance of price and therefore changes in price will be less informative of the underlying primitives of the model. Moreover, the incentive constraints of the equilibrium strategy in the first period depend on the value of κ_i.[12] This fixed point problem is expressed in the single-variable equation in Proposition 1. Importantly, since pricing strategies are not observed, κ_i is not affected by firm i's selection of price; it is determined by the pricing strategy the firm is believed to be following.

Proposition 1. *There exists a unique symmetric equilibrium in linear pricing strategies. The equilibrium strategies are determined by the value of κ in equilibrium which satisfies the following single variable equation:*

$$\kappa = \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 p_\rho^2 + \left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right) \bar{\tau}_{s,\theta}^2 p_\theta^2},$$

$$\textit{subject to } p_\theta = \frac{1}{2+\beta\kappa} \textit{ and } p_\rho = \frac{1-\left(\frac{b-e}{2b-e}\right)\beta\kappa}{2-\frac{e}{b}\bar{\tau}_{s,\rho} - \frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right)\beta^2\kappa^2},$$

where $\beta = \frac{e^2}{4b^2-e^2}$.

There are two strategic effects we can identify in the first period prices. First, due to the correlation of one cost signal and the independence of the other signal, firms may want to act more heavily on one of these signals than the other if they prefer to have their prices correlated in the first period. Additionally, firms benefit from having private information in the second period and therefore prefer to not reveal precise information about their idiosyncratic cost term. The implications of the first effect are in Proposition 2 and those of the second effect are in Proposition 3.

Proposition 2. *In equilibrium, $p_\rho < p_\theta$ when goods are complements ($e < 0$) and $p_\rho > p_\theta$ when goods are substitutes ($e > 0$). $p_\rho = p_\theta$ when markets are independent ($e = 0$).*

When $e > 0$, so that goods are substitutes, firms' first period prices are more sensitive to information on the common cost component than to information on their idiosyncratic cost component. If a firm receives a high signal on the common cost component this often implies the other firm will set a high price, increasing demand and making it optimal to further increase price. When $e < 0$, so that goods are complements, prices are strategic substitutes and will not respond strongly to the common cost signal. When $e = 0$, so that there are no cross-firm demand effects, there is no need to either adjust for the opponent's

[12] Note that a deviation does not indicate a specific misreport of marginal cost but rather an iso-information curve of feasible $s_{i,\rho}, s_{i,\theta}$. These iso-information curves depend on the value of κ_i.

price and information about each cost component affects first period prices identically. Moreover, in the monopoly case there will be no attempt to conceal information regarding cost. However, in general the information conveyed by first period prices will affect second period profits. Proposition 3 illustrates firms' incentives to not reveal too much information on their idiosyncratic cost component.

Proposition 3. *The equilibrium values of p_θ and κ are inversely related: p_θ increases when κ decreases and vice versa. Additionally, p_θ is decreasing and κ is increasing in $\bar{\tau}_{s,\theta}$, and there is a $\tau^\star$ such that for all $\bar{\tau}_{s,\rho} > \tau^\star$, κ is increasing and p_θ is decreasing in $\bar{\tau}_{s,\rho}$, and for all $\bar{\tau}_{s,\rho} < \tau^\star$, κ is decreasing and p_θ is increasing in $\bar{\tau}_{s,\rho}$. When $e > 0$, $\tau^\star > 1/2$ and when $e < 0$, $\tau^\star < 1/2$.*

When $\bar{\tau}_{s,\theta}$ is close to one, signals relatively precise information about θ_i. To maintain the strategic advantage of private information, the firm will use less of the information from a precise signal when determining first period price. If this signal is not precise, then even if the price fully reflects the information in the signal, it will still maintain private information in the second period from learning the true value of θ_i.

The presence of uncertainty on the common component of cost adds noise to the relationship between first period price and the signal on idiosyncratic cost. When this relationship is more noisy, the price reveals less information about the idiosyncratic signal, allowing the firm to use this information in its pricing decision without revealing too much information. If the signal about the common cost is relatively imprecise, $\bar{\tau}_{s,\rho}$ close to 0, then firms do not learn much information from this signal, and relatively little noise is added to this relationship. Additionally, if the signal is very precise, $\bar{\tau}_{s,\rho}$ close to 1, then when firms learn the true value of ρ in the second round, they will learn, with little error, what signal $s_{i,\rho}$ their opponents received and will be able to tease apart the noise in the pricing strategy. Therefore an intermediate level of precision $\bar{\tau}_{s,\rho}$ on signal $s_{i,\rho}$ will maximize p_θ for a given value of $\bar{\tau}_{s,\theta}$.

In general the incentives to hide idiosyncratic cost information leads firms to be less responsive to their idiosyncratic cost signal than is optimal in a one-stage game (without the informational channels implied by our two-stage model), so relaxing signal jamming incentives leads to an increased sensitivity of price to information on the idiosyncratic cost component.

3.3 Sharing industry relevant information

We now consider the effect of the firms sharing information about costs through a trade association. We assume that signals about the common cost component are shared, while those of firm's idiosyncratic shocks are not. Information shared via a trade association is that which is relevant to the production process of all firms, e.g. input costs, and firms prefer to maintain private information about idiosyncratic costs.

When firms share their signals about their common cost component they will have the same expectation about this parameter. This simplifies the two stage competition model to a generalization of the single cost component model in Mailath [1989] and Jeitschko et al. [2017]. While there are still two cost components, the informational structure is simplified so that firms posses private information about their only idiosyncratic cost components; the remaining uncertainty regarding the common cost component is common to both firms. While the optimality conditions look similar in this setting, the equilibrium pricing strategies in the first period fully reveal the private information of each firm. We briefly outline the significant differences from the previous section.[13]

In the second period the information that is available to each firm now includes $s_\rho = (s_{i,\rho}, s_{j,\rho})$. The new first order conditions are given in Lemma 5.

Lemma 5. *Firm i's optimal second period price is*

$$p_{i,2}^c = \frac{1}{2b}\left(a + bc_i + e\mathbb{E}\left[p_{j,2}^c \middle| \rho, p_1, s_\rho\right]\right).$$

Firm i's optimal first period price is

$$\begin{aligned} p_{i,1}^c = &\left(\frac{1}{2b}\right)\mathbb{E}\left[bc_i + a + ep_{j,1}^c \middle| s_\rho, s_{i,\theta}\right] \\ &+ e\left(\frac{1}{2b}\right)^2 \mathbb{E}\left[\left(a - bc_i + e\mathbb{E}\left[p_{j,2}^c \middle| \rho, s_\rho, p_1\right]\right)\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p_{j,2}^c \middle| \rho, s_\rho, p_1\right] \middle| s_\rho, s_{i,\theta}\right]. \end{aligned}$$

In a linear equilibrium, the first period price is $p_{i,1}^c = p_{0,c} + p_{\theta,c}E[\theta_i|s_{i,\theta}] + p_{\rho,c}E[\rho|s_\rho]$. Because s_ρ and $p_{i,1}$ are publicly observable, then in equilibrium, the value of $s_{i,\theta}$ can be inferred by competing firms. Therefore the expectation of each firm's cost in the second period given publicly available information is $\mathbb{E}[c_i|\rho, s_\rho, p_{i,1}] = \rho + \mathbb{E}[\theta_i|s_{i,\theta}]$, where $s_{i,\theta}$ can be determined from the first period price. Moreover, an increase in the first period price will increase this expectation by the inverse of the equilibrium coefficient $p_{\theta,c}$. The effect of firm i's first period price on firm j's second period price takes into account this informational parameter as well as effects on demand,

$$\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p_{j,2}^c \middle| \rho, p_{i,1}\right] = \frac{be}{4b^2 - e^2}\kappa^c, \text{ where } \kappa^c \equiv \frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[c_i \middle| \rho, s_\rho, p_{i,1}\right] = \frac{1}{p_{\theta,c}}$$

[13] For a more through discussion of the simplified model, see Jeitschko et al. [2017].

In the unique linear equilibrium, $p_{\theta,c}$ is strictly less than p_θ. Therefore firms use less idiosyncratic information in their first period price choice once they have shared common cost information.

Proposition 4. *In the unique equilibrium in linear pricing strategies the coefficient on idiosyncratic information is less than the corresponding coefficient in the equilibrium without information sharing:*

$$p_{\theta,c} = \frac{1-\beta}{2} \leq p_\theta.$$

This inequality is strict when $e \neq 0$. Moreover, the coefficient on common cost information is larger, $p_{\rho,c} \geq p_\rho$, when the firms are selling substitutes ($e \geq 0$).

We can similarly compare the informativeness of first period price about the underlying cost before and after the firms share information on their common cost component. Because $p_{\theta,c} \leq p_\theta$,

$$\beta\kappa = \frac{1-2p_\theta}{p_\theta} \leq \frac{1-2p_{\theta,c}}{p_{\theta,c}} = \frac{\beta}{p_{\theta,c}} = \beta\kappa^c \quad \Longleftrightarrow \quad \kappa \leq \kappa^c.$$

Following from Proposition 4, this inequality is strict when $e \neq 0$.

Once firms have shared common cost information, firms' second period prices are more responsive to the price choices in the first period. In this setting, it is easier to soften future competition and therefore firms have a greater incentive to choose a higher first period price. The increase in expected price imposes a first order negative effect on consumer welfare in the market.

Proposition 5. *Expected first period prices are higher when firms share signals about common cost information, $\mathbb{E}[p_{i,1}] \leq \mathbb{E}[p^c_{i,1}]$. Moreover, expected second period prices are the same regardless of firms sharing information or not.*

From the first order conditions in each case, it is clear that the only differences in determining the optimal price is the expected price by the competing firm in the second period and the rate at which a first period price increase by a firm affects this second period price by the competitor. It is shown that this rate of increase is higher in the case where firms share common cost information. The expectation of the second period price by the competing firm must be the same in either case. This stems from the best response function of each firm being linear in the beliefs about the competing firms costs. On average, the beliefs must be correct in equilibrium and therefore the expected price will also be the same.

4 Information acquisition

We now consider the case where firms can independently pay to improve their estimates of their individual cost components. For consistency with Section 4.2, we consider this as a problem of investing in a reputable outside research consultant: each firm pays its consultant, who returns a more or less detailed report on the state of the components of marginal cost. In our model where firms pool information, the consultant pools the research resources of the individual firms to generate a more informative signal of the state of the world; in this case, the consultant acts as a trade association. Thus the "consultant" language helps to bridge the divide between the standalone and pooled research models.

The consultant uses the payment to perform research, where greater payment improves the quality of the research. Conceptually, the precision of the firm's signals is increasing in the transfer to the consultant. As we are interested in comparative statics and not analytical solutions to the problem of information acquisition, we do not explicitly model costs in the firms' profit functions.[14] Because there are decreasing returns to informational precision (Lemmas 7 and 9), any convex cost is sufficient for our results; decreasing marginal benefits in precision point in the same direction as increasing marginal costs, and there is a finite level of precision which is optimal.[15]

Play proceeds as follows: in stage zero, firms pay consultants and consultants provide the firms with (separate) signals of their idiosyncratic and common cost components. Precisions are chosen simultaneously and are not publicly observed. In a symmetric equilibrium, the level of precision of each firm will be known prior to competition with $\tau_{i,x} = \tau_{j,x}$ for $x = \theta, \rho$. Therefore stages one and two are exactly as in Section 3, with the caveat that the information obtained in these stages does not arrive from an exogenous source but from the consultants paid in stage zero. In order for this two-part analysis to be an equilibrium in the three-stage game, an unobserved deviation in the acquisition stage must not benefit a firm in the competition stages. Application of the envelope theorem implies that a marginal change in precision would not affect the optimal strategy choice of either firm in the competition stages.

We continue our analysis of the linear pricing equilibrium obtained in Section 3, where

$$p_{i1} = p_{i0} + p_{i\theta}\mathbb{E}\left[\theta_i \middle| s_{i,\theta}\right] + p_{i,\rho}\mathbb{E}\left[\rho \middle| s_{i,\rho}\right].$$

[14] We do implicitly assume that costs are symmetric, but this is not essential.

[15] Decreasing marginal returns and increasing marginal costs frequently imply unique optima. In this case, marginal returns are determined by *believed* investment in precision while actual investment is unobservable. We discuss this at length below. Subject to believed investments, there is a unique level of investment which is optimal. In equilibrium these two must be equal.

The price coefficients p_{i0}, $p_{i\theta}$, and $p_{i\rho}$ each depend on the precisions $\tau_{i,\rho}$ and $\tau_{i,\theta}$. Firm i's stage-zero objective is

$$\max_{\tau_{i,\theta},\tau_{i,\rho}} \mathbb{E}\left[\mathbb{E}\left[\pi_{i,1}\middle| s_i\right]\right] + \mathbb{E}\left[\mathbb{E}\left[\pi_{i,2}\middle| s_i\right]\right] - c\left(\tau_{i,\theta}, \tau_{i,\rho}\right).$$

As mentioned, the precise form of the cost function is unimportant as long as it is convex, and we will ignore the costs of precision. The additive separability of the firm's profit function allows us to independently analyze the effects of precision on first and second stage profits.

Importantly, the unobservability of precision investments and the subsequent selection of prices allow for the application of the envelope theorem; then we can constrain attention to only terms which vary directly with $\tau_{i,x}$ and can ignore the effect on subsequent choice variables. Second period profits also depend on the relative informativeness of first period prices, given by κ_i, which depends on $\tau_{i,x}$. Note that deviations in $\tau_{i,x}$ do not directly affect second period profits, and enter only through κ_i and firm j's beliefs about firm i's costs. Because deviations in $\tau_{i,x}$ are not observable, starting at any putative equilibrium κ_i will remain constant even under changes in $\tau_{i,x}$. Informational precision also does not affect first stage profits except through the reduction in variance and changes in price parameters, but the latter can be ignored by application of the envelope theorem. Then our analysis of the effect of precision on the firm's expected profits can focus only on the precision terms that appear directly in profits, and can ignore all other terms.

4.1 Expected profits

First stage profits can be represented compactly in terms which vary directly with the choice of precision $\tau_{i,x}$ and composite remainder terms which vary only indirectly through $\tau_{i,x}$, and hence (by the envelope theorem) may be ignored during optimization.

Lemma 6. *There is a function $C_i : \mathbb{R}^6 \to \mathbb{R}$ varying with first-stage price coefficients and independent of τ_i such that first-stage profits are given by*

$$\mathbb{E}\left[\pi_{i,1}\right] = \left(\frac{\left(1-p_{i\theta}\right)p_{i\theta}\tau_{i,\theta}}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta}\right)b + \left(\frac{\left(1-p_{i\rho}\right)p_{i\rho}\tau_{i,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho}\right)b - \left(\frac{\left(1-p_{i\rho}\right)p_{j\rho}\tau_{i,\rho}\tau_{j,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho}\right)e + C_i.$$ [16]

First stage profits respond to precision of the two components of marginal cost in a similar way with respect to own demand (the terms postmultiplied by b), but the response differs

[16] The arguments to C_i are omitted for compactness; the equation in Lemma 6 should be written with $C_i(p_{i0}, p_{i\theta}, p_{i\rho}, p_{j0}, p_{j\theta}, p_{j\rho})$. As discussed earlier the dropping of these arguments is unimportant, as C_i does not vary directly with τ_i and thus will be ignored when optimizing over the choice of precision.

with respect to cross-firm demand (the term postmultiplied by e). To a first approximation, the precision of the informational signals affects the opponent's payoffs only through information on the common cost component; increasing this precision will increase the correlation in first-period prices, and decreasing this precision will reduce the correlation in first-period prices.

Lemma 7. *The marginal changes in first-stage profits with respect to the precision $\tau_{i,\theta}$ and $\tau_{i,\rho}$ are*

$$\frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}\right] = \left(\frac{(1 - p_{i\theta})\, p_{i\theta}}{(\tau_{i,\theta} + \tau_{\theta})^2} \right) b; \tag{1}$$

$$\frac{\partial}{\partial \tau_{i,\rho}} \mathbb{E}\left[\pi_{i,1}\right] = \left(\frac{(1 - p_{i\rho})\, p_{i\rho}}{(\tau_{i,\rho} + \tau_{\rho})^2} \right) (b - \bar{\tau}_{j,\rho} e). \tag{2}$$

Because all involved terms are positive, it is immediate that when goods are complements (so that $e < 0$) first stage profits respond more strongly to precision on the common cost component than to the idiosyncratic cost component (holding all else fixed); when goods are substitutes (so that $e > 0$) first stage profits respond more strongly to precision on the idiosyncratic cost component than to the common cost component. As previously discussed, when goods are independent (so that $e = 0$) we have $p_{i\theta} = p_{i\rho}$, implying that first-stage profits respond identically to information on either cost component. If the precisions of the underlying variables θ_i and ρ are not identical, first-stage profits will respond more strongly to increases in precision of the signal of the less precise underlying variable.

Importantly marginal profit of a deviation in precision is completely determined by the deviation's effect on first period profits. This follows from three observations. First, the envelope theorem implies that precision's effect on profits via its effect on prices can be ignored. Since prices are optimally selected subject to precision the derivative of profit with respect to prices is zero, and this term vanishes. Second, precision has no effect on the firm's own inferences in the second period. Because cost parameters are perfectly observed, if precision did affect the firm's inferences it would need to affect the conditional expectation of its opponent's costs given first period prices and the known value of the common cost parameter ρ. Since investment is unobservable the opposing firm's strategy is unchanged, and all inferences remain unchanged. Third, and relatedly, investment in precision does not affect the opposing firm's second-period pricing strategy. Because precision is unobservable the opposing firm has no way of knowing that its inferences are incorrect and will not alter its strategy.[17]

[17] Note that while maginal deviations in investment in precision cannot affect second-period profits, differ-

Proposition 6. *Conditional on $\tau_{i,\rho}$, all equilibria in the private information acquisition model have a unique and symmetric choice of $\tau_{i,\theta}$.*

Proof. We have already seen (in Proposition 1) that conditional on precision there is a unique equilibrium in the pricing game. With information acquisition, opponent precision is unobservable. Fixing believed equilibrium inference $\hat{\kappa}$, it is the case that the firm's pricing *strategy* is independent of its level of precision investment: prices are functions of the conditional expectations of cost parameters, and the entire effect of precision is captured in the conditional expectation. Lemma 6 implies that there are strictly decreasing marginal returns to precision, and since the cost of precision is convex it follows that there is a unique level of precision, and hence unique pricing strategy, taking as given believed equilibrium inference $\hat{\kappa}$.

Then equilibrium will be nonunique only if there are two equilibrium inference levels $\hat{\kappa}$ and $\hat{\kappa}'$, leading to distinct precision and pricing decisions. Suppose $\hat{\kappa}$ is an equilibrium belief, and $\hat{\kappa}' > \hat{\kappa}$. By Proposition 3, $p'_\theta < p_\theta$. Since $p_\theta < 1/2$ in any equilibrium, Lemma 7 implies that the marginal return to precision is lower under belief $\hat{\kappa}'$ than under belief $\hat{\kappa}$. Then $\tau'_{i,\theta} < \tau_{i,\theta}$. Then Proposition 3 implies that $\hat{\kappa}' < \hat{\kappa}$, a contradiction. Then there is a unique level of informativeness $\hat{\kappa}$ and unique choice of $\tau_{i,\theta}$. □

Corollary 1. *$\tau_{i,\theta}$ is increasing in $\tau_{i,\rho}$ when $\bar{\tau}_{i,\rho}$ is close to zero, and decreasing in $\tau_{i,\rho}$ when $\bar{\tau}_{i,\rho}$ is close to one.*

Corollary 1 follows from equation (1). Proposition 3 states that p_θ is decreasing in $\tau_{i,\rho}$ when $\bar{\tau}_{i,\rho}$ is large, and increasing in $\tau_{i,\rho}$ when $\bar{\tau}_{i,\rho}$ is small. Then since $p_\theta \leq 1/2$ for all precision levels, the marginal utility gain from a small increase in $\tau_{i,\rho}$ decreases when $\bar{\tau}_{i,\rho}$ is large and increases when $\bar{\tau}_{i,\rho}$ is small. Then improved precision on the common component leads to decreased precision on the idiosyncratic component when common component precision is already high, and leads to increased precision on the idiosyncratic component when common component precision is low.

Intuitively, when common component precision is high it is difficult for the firm to mask its private signal. Further increasing the precision on the common component makes this even more difficult, and reduces the returns of acquiring private information as the opponent will learn of this information prior to the second period. In the other direction, when the signal on the common component is relatively imprecise, increasing this precision captures some available profits through improved information — absent signaling incentives, reducing variance is good for the firm. Then since the signal is still relatively imprecise, the firm can

ent levels of believed investment will typically generate different second-period profits.

still confound its opponent by the use of signaling in its pricing strategy and the information on the idiosyncratic term will be more valuable in the second period

4.2 Sharing industry relevant information

We now develop our model of information acquisition so that there is a single technology responsible for improving information on the common cost component. Consider, for example, a research consultant that takes payment in exchange for producing reports. If products rely on a common input factor forecasts of the market for this input are equally valuable to either firm. The consultant acts as a pooling resource, producing forecasts from the total payments that it receives from either firm. At a high level this is akin to the consultant releasing a public whitepaper containing its market forecasts, or a trade association producing an article in an industry journal.

To find the optimal precision of signal we find the effect of an increase in precision of both common and private cost components on each of the two periods of competition. Since an increase in precision does not affect the expected value of each component of the cost, it will not affect the expected prices of either firm in either period of competition. Therefore the precision will only effect the variance and correlation of firms' prices in each period.

Increasing the precision of the private cost component of a firm will increase the variance of that firm's first period price choice and lead to higher profits in that period. On the other hand, since all information will be revealed to the competing firm through the first period price, this will increase the information available to that firm in the second period. When the firms sell substitutes or complements, this will increase the correlation of prices in this round which leads to lower second period profits. For a more thorough discussion of this point see Jeitschko et al (2017).

Increasing the precision of the common cost component will both increase the variance of first period price choices and the correlation of strategies. Again, the increase in variance will have a positive effect on ex-ante expected payoffs while a increased correlation will decrease these payoffs. The level of precision will have no affect on the second period profits of either firm, as each firm will know the true value of ρ at that time.

Lemma 8. *There is a constant $C_{i,c}$ which is independent of τ_i such that first-stage profits are given by*

$$\mathbb{E}\left[\pi_{i,1}\left(s_p, s_{i,\theta}\right)\middle|\, \tau_{\varepsilon,\rho}, \tau_{i,\theta}\right] = \left(\frac{\left(1-p_\theta\right) p_\theta \tau_{i,\theta}}{\left(\tau_\theta + \tau_{i,\theta}\right)\tau_\theta}\right) b + \left(\frac{\left(1-p_\rho\right)\tau_{i,\rho}}{\left(\tau_\rho + \tau_{i,\rho}\right)\tau_\rho}\right)(b-e) + C_{i,c}.$$

Lemma 9. *The marginal changes in first-stage profits with respect to the precision* $\tau_{i,\theta}$ *and* $\tau_{c,\rho}$ *are*

$$\frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}^{c}\right] = \left(\frac{(1-p_{\theta,c})\, p_{\theta,c}}{(\tau_{i,\theta}+\tau_{\theta})^2}\right) b,$$
$$\frac{\partial}{\partial \tau_{c,\rho}} \mathbb{E}\left[\pi_{i,1}^{c}\right] = \left(\frac{(1-p_{\rho,c})\, p_{\rho,c}}{(\tau_{c,\rho}+\tau_{\rho})^2}\right)(b-e)\,.$$

Lemma 10 (Unique equilibrium)**.** *There is a unique equilibrium in the pooled information model.*

Proof. This follows directly from Lemma 9 and Proposition 4, which states that the unique equilibrium choice of $p_{\theta,c}$ and $p_{\rho,c}$ do not depend on $\tau_{i,\theta}$ and $\tau_{c,\rho}$. □

Proposition 7 (Relative value of information)**.** *The marginal benefit of acquiring additional information in the private cost component is lower when information about the common cost component is gathered by the trade association compared to when common cost information is acquired privately by the two firms,*

$$\frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}\right] > \frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}^{c}\right].$$

Proof. Comparing marginal benefits in the first period

$$\frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}\right] = \left(\frac{(1-p_{i\theta})\, p_{i\theta}}{(\tau_{i,\theta}+\tau_{\theta})^2}\right) b, \text{ and } \frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}^{c}\right] = \left(\frac{(1-p_{\theta,c})\, p_{\theta,c}}{(\tau_{i,\theta}+\tau_{\theta})^2}\right) b.$$

From Proposition 4, $p_{i,\theta}^{c} < p_{i,\theta}^{*} < 1/2$ which implies directly that $\frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}\left[\pi_{i,1}\right] > \frac{\partial}{\partial \tau_{i,\theta}} \mathbb{E}[\pi_{i,1}^{c}]$. □

Since the marginal benefit of an increase in precision on each stage of profits is lower compared to when there is no informational middleman then firms will gather less information about their private cost component. This may lead to worse outcomes for the firms.

For the common cost component, increased correlation of the signal reduces the value of this signal in the first stage of competition. Moreover, the level of precision has no effect on second stage profits, as the true value of ρ is learned by both firms. Without the informational middleman the precision still affects the second stage due to its signal jamming effect. The direction of this effect is less clear, as increase in the precision of this signal, means that the signal will be closer to the true value and the other firm will become more informed about the signal when they learn ρ. However, a completely uninformative signal does not signal jam at all.

5 Conclusion

We generalize a standard dynamic pricing competition model to allow for uncertainty in common cost and private cost parameters. We characterize the unique symmetric linear equilibrium of this model and use this to examine the incentives of firms to acquire and subsequently share information in this setting. We show that the incentive of firms to acquire additional information about private cost parameters is reduced when firms in the industry pool resources to acquire information about common costs. On the other hand, coordination of information acquisition on idiosyncratic cost terms can alleviate inefficiently low information acquisition. Therefore, the welfare effects of coordination on information between firms depends crucially on the type of information that is acquired.

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A Proofs

Proof of Lemma 2

From Lemma 1,

$$p^\star_{i,2} = \frac{1}{2b}\left(a + bc_i + e\mathbb{E}\left[p^\star_{j,2}\middle|\,\rho, p_{j,1}, p_{i,1}\right]\right).$$

Therefore the expected price of firm j from the perspective of firm i given the public information in the second period is

$$\begin{aligned}\mathbb{E}[p^\star_{j,2}|\rho, p_{i,1}, p_{j,1}] &= \frac{1}{2b}\left(a + \left(\mathbb{E}\left[c_j|\,\rho, p_{j,1}, p_{i,1}\right]\right) b + e\mathbb{E}[p^\star_{i,2}|\rho, p_1]\right) \\ &= \frac{1}{2b}\left(a + b\mathbb{E}\left[c_j|\,\rho, p_{j,1}, p_{i,1}\right]\right) + \frac{e}{4b^2}\left(a + b\mathbb{E}\left[c_i|\,\rho, p_{j,1}, p_{i,1}\right] + e\mathbb{E}[p^\star_{j,2}|\rho, p_1]\right).\end{aligned}$$

Note that firm j's beliefs on firm i's price can only be conditioned on public observables (ρ and first-period prices p_1) and firm i learns nothing [additional] by deviating in the first period, it must be that firm i's expectation of firm j's expectation of firm i's price is equal to the expectation of firms i's price conditioning only on public observables and equilibrium behavior. This leaves

$$\mathbb{E}[p^{\star}_{j,2}|\rho, p_{i,1}, p_{j,1}] = \frac{2b}{4b^2 - e^2}\left[(a + b\mathbb{E}\left[c_j|\, \rho, p_{j,1}\right]) + \frac{e}{2b}\left(a + b\mathbb{E}\left[c_i|\, \rho, p_{i,1}\right]\right)\right].$$

Proof of Lemma 4

Define $\bar{\tau}_{s,x} = \frac{1/\sigma^2_{s,x}}{1/\sigma^2_x + 1/\sigma^2_{s,x}}$ and for $x \in \{\theta_i, \theta_j, \rho\}$. Then, in a linear (first stage) equilibrium,

$$\begin{pmatrix} c_i \\ \rho \\ p_{i,1} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \bar{\tau}_{s,\theta} p_\theta & \bar{\tau}_{s,\rho} p_\rho \end{pmatrix} \begin{pmatrix} \rho \\ c_i \\ s_{i,\theta} \\ s_{i,\rho} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p_0 + (1 - \bar{\tau}_{s,\theta})\,\mu_\theta p_\theta + (1 - \bar{\tau}_{s,\rho})\,\mu_\rho p_\rho \end{pmatrix}$$

$$\sim N\left(\begin{pmatrix} \mu_\rho + \mu_\theta \\ \mu_\rho \\ p_0 + p_\theta \mu_\theta + p_\rho \mu_\rho \end{pmatrix}, \begin{pmatrix} \sigma^2_c & \sigma^2_\rho & \bar{\tau}_{s,\theta}\sigma^2_\theta p_\theta + \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho \\ \sigma^2_\rho & \sigma^2_\rho & \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho \\ \bar{\tau}_{s,\theta}\sigma^2_\theta p_\theta + \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho & \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho & \bar{\tau}^2_{s,\theta}\left(\sigma^2_{s,\theta} + \sigma^2_\theta\right) p^2_\theta + \bar{\tau}^2_{s,\rho}\left(\sigma^2_{s,\rho} + \sigma^2_\rho\right) p^2_\rho \end{pmatrix}\right).$$

It follows that the conditional expectation of c_j given ρ and $p_{j,1}$ is

$$\begin{aligned}
\mathbb{E}\left[c_i|\, \rho, p_{i,1}\right] &= (\mu_\rho + \mu_\theta) + \Sigma_{12}\Sigma^{-1}_{22}\left(\begin{pmatrix} \rho \\ p_{i,1} \end{pmatrix} - \begin{pmatrix} \mu_\rho \\ p_0 + p_\theta\mu_\theta + p_\rho\mu_\rho \end{pmatrix}\right), \\
\Sigma_{12} &= \begin{pmatrix} \sigma^2_\rho & \bar{\tau}_{s,\theta}\sigma^2_\theta p_\theta + \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho \end{pmatrix}, \\
\Sigma_{22} &= \begin{pmatrix} \sigma^2_\rho & \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho \\ \bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho & \bar{\tau}^2_{s,\theta}\left(\sigma^2_{s,\theta} + \sigma^2_\theta\right) p^2_\theta + \bar{\tau}^2_{s,\rho}\left(\sigma^2_{s,\rho} + \sigma^2_\rho\right) p^2_\rho \end{pmatrix}.
\end{aligned}$$

Letting $\Sigma_{12}\Sigma^{-1}_{22} = (m_1 \;\; m_2)$, it follows that $\kappa_i \equiv \partial\mathbb{E}[\mathbf{c}_i|\rho, p_{i,1}]/\partial p_{i,1} = m_2$; in particular, we only need to care about the right-hand column of Σ^{-1}_{22},

$$\Sigma^{-1}_{22} = \frac{1}{|\Sigma_{22}|}\begin{pmatrix} \cdot & -\bar{\tau}_{s,\rho}\sigma^2_\rho p_\rho \\ \cdot & \sigma^2_\rho \end{pmatrix}.$$

It follows that

$$\kappa_i = \frac{\bar{\tau}_{s,\theta}\sigma^2_\theta\sigma^2_\rho p_\theta}{|\Sigma_{22}|} = \frac{\bar{\tau}_{s,\theta}\sigma^2_\theta\sigma^2_\rho p_\theta}{\bar{\tau}^2_{s,\theta}\sigma^2_{s,\theta}\sigma^2_\rho p^2_\theta + \bar{\tau}^2_{s,\rho}\sigma^2_\rho(\sigma^2_{s,\rho} + \sigma^2_\rho)p^2_\rho - \bar{\tau}^2_{s,\rho}\sigma^4_\rho p^2_\rho} = \frac{\sigma^2_\theta\bar{\tau}_{s,\theta}p_\theta}{\sigma^2_{\varepsilon,\rho}\bar{\tau}^2_{s,\rho}p^2_\rho + \left(\sigma^2_\theta + \sigma^2_{\varepsilon,\theta}\right)\bar{\tau}^2_{s,\theta}p^2_\theta}.$$

Then

$$\frac{\partial}{\partial p}\mathbb{E}[p^\star_{j,2}|\rho,p_1]=\frac{be}{4b^2-e^2}\kappa=\frac{be}{4b^2-e^2}\left(\frac{\sigma^2_\theta\bar{\tau}_{s,\theta}p_\theta}{\sigma^2_{s,\rho}\bar{\tau}^2_{s,\rho}p^2_\rho+\left(\sigma^2_\theta+\sigma^2_{s,\theta}\right)\bar{\tau}^2_{s,\theta}p^2_\theta}\right).$$

Proof of Proposition 1

The first order condition is given by:

$$p^\star_{i,1}=\frac{1}{2b}\mathbb{E}\left[bc_i+a+e\hat{p}_{j,1}+\frac{e}{2b}\left(a-bc_i+e\mathbb{E}\left[p^\star_{j,2}\middle|\rho,p^\star_{i,1},\hat{p}_{j,1}\right]\right)\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p^\star_{j,2}\middle|\rho,p^\star_{i,1},\hat{p}_{j,1}\right]\middle|s_{i,\rho},s_{i,\theta}\right],$$

where

$$\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p^\star_{j,2}\middle|\rho,p^\star_{i,1},\hat{p}_{j,1}\right]=\frac{be}{4b^2-e^2}\kappa_i$$

and

$$\mathbb{E}\left[\mathbb{E}\left[p^\star_{j,2}\middle|\rho,p_1\right]\middle|s_{i,\rho},s_{i,\theta}\right]=\frac{2b}{4b^2-e^2}\left[\left(a+b\mathbb{E}\left[\mathbb{E}\left[c_j|\rho,p_{j,1}\right]|s_{i,\rho},s_{i,\theta}\right]\right)+\frac{e}{2b}\left(a+b\mathbb{E}\left[\mathbb{E}\left[c_i|\rho,p_{i,1}\right]|s_{i,\rho},s_{i,\theta}\right]\right)\right].$$

In a linear equilibrium, the random variables $(c_k,\rho,p_{k,1})$ are jointly normal and the conditional expectation of cost in the second period is

$$\begin{aligned}\mathbb{E}\left[c_k|\rho,p_{k,1}\right]&=(\mu_\rho+\mu_\theta)+\Sigma_{12}\Sigma_{22}^{-1}\left(\begin{pmatrix}\rho\\p_{k,1}\end{pmatrix}-\begin{pmatrix}\mu_\rho\\p_0+p_\theta\mu_\theta+p_\rho\mu_\rho\end{pmatrix}\right)\\&=(\mu_\rho+\mu_\theta)+(1-\kappa_k\bar{\tau}_{s,\rho,k}p_\rho)(\rho-\mu_\rho)+(p_{k,1}-(p_0+p_\rho\mu_\rho+p_\theta\mu_\theta))\kappa_k.\end{aligned}$$

Therefore the expectation of this conditional expectation given the signals available in the first period are

$$\mathbb{E}\left[\mathbb{E}\left[c_j|\rho,p_{j,1}\right]|s_{i,\rho},s_{i,\theta}\right]=\mathbb{E}\left[\rho|s_{i,\rho}\right]+\mu_\theta \text{ and}$$
$$\mathbb{E}\left[\mathbb{E}\left[c_i|\rho,p_{i,1}\right]|s_{i,\rho},s_{i,\theta}\right]=\mathbb{E}\left[\rho|s_{i,\rho}\right]+\mu_\theta+\kappa_ip_\rho(1-\bar{\tau}_{s,\rho,i})\left(\mathbb{E}\left[\rho|s_{i,\rho}\right]-\mu_\rho\right)+\kappa_ip_\theta\left(\mathbb{E}\left[\theta_i|s_{i,\theta}\right]-\mu_\theta\right),$$

and

$$\begin{aligned}\mathbb{E}\left[\mathbb{E}\left[p^\star_{j,2}\middle|\rho,p_1\right]\middle|s_{i,\rho},s_{i,\theta}\right]&=\frac{2b}{4b^2-e^2}\left(a+b\left(\mu_\theta+\mathbb{E}\left[\rho|s_{i,\rho}\right]\right)\right)\\&+\frac{e}{4b^2-e^2}\left(a+b\left(\mu_\theta+\kappa_ip_\theta\left(\mathbb{E}\left[\theta_i|s_{i,\theta}\right]-\mu_\theta\right)+\mathbb{E}\left[\rho|s_{i,\rho}\right]+\kappa_ip_\rho(1-\bar{\tau}_{s,\rho,i})\left(\mathbb{E}\left[\rho|s_{i,\rho}\right]-\mu_\rho\right)\right)\right).\end{aligned}$$

The first order condition becomes

$$
\begin{aligned}
2bp_{i,1}^{\star} =& \mathbb{E}\left[bc_i + a + ep_{j,1} + \frac{e}{2b}\left(a - bc_i + e\mathbb{E}\left[p_{j,2}^{\star} \middle| \rho, p_{i,1}^{\star}, \hat{p}_{j,1}\right]\right) \frac{\partial}{\partial p_{i,1}} \mathbb{E}\left[p_{j,2}^{\star} \middle| \rho, p_{i,1}^{\star}, \hat{p}_{j,1}\right] \middle| s_{i,\rho}, s_{i,\theta}\right] \\
=& a + e\mathbb{E}\left[p_{j,1} \middle| s_{i,\rho}\right] + b\mathbb{E}\left[\mathbf{c}_i \middle| s_{i,\rho}, s_{i,\theta}\right] + b\beta\kappa_i \left(\frac{a}{2b-e} + \frac{\left(\mu_\theta + \mathbb{E}\left[\rho \middle| s_{i,\rho}\right]\right) be}{4b^2 - e^2} \right. \\
&\left. - \frac{\mathbb{E}\left[\mathbf{c}_i \middle| s_{i,\rho}, s_{i,\theta}\right]}{2} + \frac{\beta}{2}\left(\mu_\theta + \kappa_i p_\theta \left(\mathbb{E}\left[\theta \middle| s_{i,\theta}\right] - \mu_\theta\right) + \mathbb{E}\left[\rho \middle| s_{i,\rho}\right] + \kappa_i p_\rho \left(1 - \bar{\tau}_{s,\rho,i}\right)\left(\mathbb{E}\left[\rho \middle| s_{i,\rho}\right] - \mu_\rho\right)\right)\right)
\end{aligned}
$$

Lastly, note that

$$
\begin{aligned}
\mathbb{E}\left[p_{j,1} \middle| s_{i,\rho}\right] &= p_0 + \mu_\theta p_\theta + \mathbb{E}\left[\mathbb{E}\left[\rho \middle| s_{j,\rho}\right] \middle| s_{i,\rho}\right] p_{\rho,j} \\
&= p_0 + \mu_\theta p_\theta + \mathbb{E}\left[\bar{\tau}_{s,\rho,j} s_{j,\rho} + \left(1 - \bar{\tau}_{s,\rho,j}\right)\mu_\rho \middle| s_{i,\rho}\right] p_{\rho,j} \\
&= p_0 + \mu_\theta p_\theta + \bar{\tau}_{s,\rho,j}\mathbb{E}\left[\rho \middle| s_{i,\rho}\right] p_{\rho,j} + \left(1 - \bar{\tau}_{s,\rho,j}\right)\mu_\rho p_{\rho,j}.
\end{aligned}
$$

Matching coefficients in the first order condition in Lemma 3, price coefficients satisfy the following equalities in any linear equilibrium,

$$
\begin{aligned}
2bp_0 &= a + \left(p_{0,j} + \mu_\theta p_{\theta,j} + \left(1 - \bar{\tau}_{s,\rho,j}\right)\mu_\rho p_{\rho,j}\right) e + \kappa_i b\beta \left(\frac{a}{2b-e} + \frac{\mu_\theta be}{4b^2 - e^2}\right) \\
&\quad + \frac{\kappa_i b\beta^2}{2}\left(\left(1 - \kappa_i p_\theta\right)\mu_\theta - \left(1 - \bar{\tau}_{s,\rho,i}\right)\kappa_i \mu_\rho p_\rho\right), \\
2bp_\theta &= b + \kappa b\beta \left(-\frac{1}{2} + \frac{1}{2}\beta\kappa p_\theta \right), && (3) \\
2bp_\rho &= e\bar{\tau}_{s,\rho,j} p_{\rho,j} + b + \kappa b\beta \left(\frac{be}{4b^2 - e^2} - \frac{1}{2} + \frac{\beta}{2}\left(1 + \left(1 - \bar{\tau}_{s,\rho,i}\right)\kappa p_\rho\right)\right), && (4) \\
\kappa &= \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 p_\rho^2 + \left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right)\bar{\tau}_{s,\theta}^2 p_\theta^2}. && (5)
\end{aligned}
$$

Equations (3) and (4) may be equivalently represented as quadratics in $\beta\kappa$, and solving for equilibrium amounts to equating the roots of two quadratic equations. The equations can be reduced to a system of two equations by explicitly solving the p_θ equation.

$$
\begin{aligned}
2bp_\theta &= b + \frac{\kappa be^2}{4b^2 - e^2}\left(-\frac{1}{2} + \frac{1}{2}\left(\frac{e^2}{4b^2 - e^2}\right)\kappa p_\theta \right). \\
\Rightarrow 0 &= p_\theta (\beta\kappa)^2 - \beta\kappa + 2(1 - 2p_\theta).
\end{aligned}
$$

From equation (3),

$$
\beta\kappa = \frac{1}{2p_\theta}\left(1 \pm \sqrt{1 - 8\left(1 - 2p_\theta\right)p_\theta}\right) = \frac{1}{2p_\theta}\left(1 \pm \sqrt{\left(1 - 4p_\theta\right)^2}\right) = \frac{1 \pm \left(1 - 4p_\theta\right)}{2p_\theta}.
$$

There are two solutions to this quadratic equation: $\beta\kappa = 2$ and $\beta\kappa = (1 - 2p_\theta)/p_\theta$. The second order condition is given by

$$-2b + \frac{be^4}{2(4b^2 - e^2)^2}\kappa^2 < 0$$

This simplifies to $-2b + b(\beta\kappa)^2/2 < 0$. When $\beta\kappa = 2$ then the left hand side equals zero, so the second order condition is not satisfied. When $\beta\kappa = (1 - 2p_\theta)/p_\theta$, then $p_\theta = \frac{1}{2+\beta\kappa}$, and the second order condition becomes $p_\theta > 1/6$ which is satisfied in equilibrium.

Solving Equation (4) for p_ρ in a symmetric equilibrium[18] yeilds

$$p_\rho = \frac{b - b\beta\kappa\left(\frac{b-e}{2b+e}\right)}{2b - e\bar{\tau}_{s,\rho} - \frac{1}{2}b\beta^2\kappa^2(1 - \bar{\tau}_{s,\rho})}$$

Now, focusing attention on κ, solving equations (3), (4), and (5), and therefore finding an equilibrium reduces to solving the following single-variable equation,

$$\begin{aligned} &\underbrace{(2+\beta\kappa)^2\left(1 - \beta\kappa\left(\frac{b-e}{2b-e}\right)\right)^2 \sigma_{s,\rho}^2\bar{\tau}_{s,\rho}^2\kappa}_{\text{LHS}(\kappa)} \\ &= \underbrace{\left((2b - \bar{\tau}_{s,\rho}e) - \frac{1}{2}(1 - \bar{\tau}_{s,\rho})\, b\beta^2\kappa^2\right)^2 (2 - (1-\beta)\,\kappa)\,\sigma_\theta^2\bar{\tau}_{s,\theta}}_{\text{RHS}(\kappa)}\,. \end{aligned} \tag{6}$$

To see that an equilibrium exists, let $\bar{\kappa} = 2/(1-\beta)$ be the maximum feasible value of κ and note that $\text{LHS}(0) = 0$ and $\text{RHS}(0) > 0$, and $\text{LHS}(\bar{\kappa}) > 0$ and $\text{RHS}(\bar{\kappa}) = 0$. Since both sides of the equation are continuous in κ, there exists a κ that solves the equation, and this κ will determine the linear pricing parameters p_0, p_θ, and p_ρ.

Showing uniqueness is more involved. We first show that RHS is decreasing. We then show that either LHS is increasing, or it is increasing and then decreasing. Where LHS is decreasing, it is concave and RHS is convex. Since $\text{LHS}(\bar{\kappa}) > \text{RHS}(\bar{\kappa})$ when $\bar{\tau}_{s,\rho} > 0$, there is a unique crossing point whenever $\bar{\tau}_{s,\rho} > 0$. When $\bar{\tau}_{s,\rho} = 0$, LHS is identically 0 for all κ, so there is a unique crossing point at $\kappa = \bar{\kappa}$. In either case, there is a unique feasible value of κ that solves the equilibrium sufficient condition.

First, RHS is decreasing. This can be observed directly, and calculus is not necessary. By inequalities (12) and (16), $1-\beta \in [0, 2/3]$ and $\kappa \in [0, 3]$, so $2-(1-\beta)\kappa \geq 0$ is decreasing. By inequality (18), $\beta\kappa \in [0, 1]$, so $(2b - \bar{\tau}_{s,\rho}e) - (1 - \bar{\tau}_{s,\rho})b\beta^2\kappa^2/2 > 0$ is decreasing. Then RHS is the product of two decreasing and positive functions, and is itself decreasing.

[18] Here we impose both $p_{\rho,j} = p_\rho$ and $\bar{\tau}_{s,\rho,i} = \bar{\tau}_{s,\rho,j}$.

Second, LHS is either increasing, or is increasing and then decreasing. The first derivative of LHS is

$$\begin{aligned}\frac{\partial \mathrm{LHS}}{\partial \kappa} &= 2\beta\left(2+\beta\kappa\right)\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)^2 \sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 \kappa \\ &\quad - 2\beta\left(\frac{b-e}{2b-e}\right)\left(2+\beta\kappa\right)^2\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 \kappa \\ &\quad + \left(2+\beta\kappa\right)^2\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)^2 \sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 .\end{aligned}$$

Factoring out common positive terms gives

$$\begin{aligned}\frac{\partial \widetilde{\mathrm{LHS}}}{\partial \kappa} &= 2\beta\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)\kappa - 2\beta\left(\frac{b-e}{2b-e}\right)\left(2+\beta\kappa\right)\kappa + \left(2+\beta\kappa\right)\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right) \\ &= 2 + \left(2\beta - 4\beta\left(\frac{b-e}{2b-e}\right) + \beta - 2\beta\left(\frac{b-e}{2b-e}\right)\right)\kappa \\ &\quad + \left(-2\beta^2\left(\frac{b-e}{2b-e}\right) - 2\beta^2\left(\frac{b-e}{2b-e}\right) - \beta^2\left(\frac{b-e}{2b-e}\right)\right)\kappa^2 \\ &= 2 + \left(3 - 6\left(\frac{b-e}{2b-e}\right)\right)\beta\kappa - 5\left(\frac{b-e}{2b-e}\right)\beta^2\kappa^2 . \end{aligned} \tag{7}$$

Then the sign of $\partial \mathrm{LHS}/\partial\kappa$ is determined by the sign of $\partial\widetilde{\mathrm{LHS}}/\partial\kappa$, which is a negative quadratic in κ. Since $\mathrm{LHS}(0) = 0$ and LHS is positive, it follows that either LHS is increasing, or it is increasing and then decreasing.

When LHS is increasing and then decreasing, its inflection point $\kappa^{\perp}$ will be given by one of the zeros of equation (7).[19] The quadratic equation gives these zeros as

$$\begin{aligned}\beta\kappa^{\perp} &= \frac{2b-e}{10\left(b-e\right)}\left(\left(3-\left(\frac{b-e}{2b-e}\right)\right) \pm \sqrt{\left(3-6\left(\frac{b-e}{2b-e}\right)\right)^2 + 40\left(\frac{b-e}{2b-e}\right)}\right) \\ &= \frac{1}{10\left(b-e\right)}\left(\left(3\left(2b-e\right) - 6\left(b-e\right)\right) \pm \sqrt{\left(3\left(2b-e\right)-6\left(b-e\right)\right)^2 + 40\left(b-e\right)\left(2b-e\right)}\right) \\ &= \frac{1}{10\left(b-e\right)}\left(3e \pm \sqrt{\left(3e\right)^2 + 40\left(2b^2 - 3be + e^2\right)}\right).\end{aligned}$$

Since $\kappa \geq 0$ and the radicand is weakly larger than $(3e)^2$, only the "plus" solution to the

[19] We analyze equation (7) as a quadratic in $\beta\kappa$. Since we will show that $\beta\kappa$ is decreasing in r_{eb} and β is increasing in r_{eb}, it follows that κ is decreasing in r_{eb}.

quadratic is valid. Letting $r_{eb} = e/b$ and treating this as a quadratic in $\beta\kappa$, this is

$$\beta\kappa^{\perp} = \frac{1}{10 - 10r_{eb}} \left(3r_{eb} + \sqrt{80 - 120r_{eb} + 49r_{eb}^2} \right).$$

Taking the derivative of $\kappa^{\perp}$ with respect to r_{eb} gives

$$\begin{aligned} \frac{\partial\beta\kappa^{\perp}}{\partial r_{eb}} = \left(\frac{1}{10 - 10r_{eb}} \right)^2 \Bigg(\left(3 + \frac{49r_{eb} - 60}{\sqrt{80 - 120r_{eb} + 49r_{eb}^2}} \right) (10 - 10r_{eb}) \\ +10 \left(3r_{eb} + \sqrt{80 - 120r_{eb} + 49r_{eb}^2} \right) \Bigg). \end{aligned} \tag{8}$$

We want to show that $\kappa^{\perp}$ is minimized when $e = -b$, so we check that $\partial\beta\kappa^{\perp}/\partial r_{eb} > 0$; it is sufficient to check the numerator of equation (8).

$$\begin{aligned} 0 &\gtrless \left(3 + \frac{49r_{eb} - 60}{\sqrt{80 - 120r_{eb} + 49r_{eb}^2}} \right) (10 - 10r_{eb}) \\ &\quad + 10 \left(3r_{eb} + \sqrt{80 - 120r_{eb} + 49r_{eb}^2} \right) \\ \iff 0 &\gtrless \left(3\sqrt{80 - 120r_{eb} + 49r_{eb}^2} + 49r_{eb} - 60 \right) (1 - r_{eb}) \\ &\quad + \left(3r_{eb}\sqrt{80 - 120r_{eb} + 49r_{eb}^2} + 80 - 120r_{eb} + 49r_{eb}^2 \right) \\ \iff &- 3\sqrt{80 - 120r_{eb} + 49r_{eb}^2} \\ &\gtrless (49r_{eb} - 60)(1 - r_{eb}) + \left(80 - 120r_{eb} + 49r_{eb}^2\right) = 20 - 11r_{eb}. \end{aligned}$$

Since the left-hand side of the final inequality is always negative and the right-hand side is always positive, it is the case that $\partial\beta\kappa^{\perp}/\partial r_{eb} > 0$. Then $\beta\kappa^{\perp}$ is minimized at $r_{eb} = -1$, or $e = -b$. This gives

$$\beta\kappa^{\perp} = \frac{1}{20} \left(-3 + \sqrt{80 + 120 + 49} \right) \quad \Longrightarrow \quad \kappa^{\perp} = \frac{3}{20} \left(-3 + \sqrt{249} \right) \approx 1.917 \geq \frac{3}{2}.$$

Then when LHS is decreasing, it is only decreasing for $\kappa > 3/2$.

We now show that where LHS is decreasing, it is concave. Placing the common positive terms back into equation (7) gives

$$\frac{\partial \text{LHS}}{\partial\kappa} \propto \left(2 + \left(3 - 6 \left(\frac{b - e}{2b - e} \right) \right) \beta\kappa - 5 \left(\frac{b - e}{2b - e} \right) \beta^2\kappa^2 \right) (2 + \beta\kappa) \left(1 - \beta\kappa \left(\frac{b - e}{2b - e} \right) \right). \tag{9}$$

Two points are of note: first, equation (7) implies $[\partial\widetilde{\text{LHS}}/\partial\kappa]|_{e=0} > 0$, so if LHS is decreasing our earlier result showing that $\kappa^{\perp}$ is increasing in r_{eb} implies that $e < 0$. Second, the left two terms in relationship 9 reduce to

$$(2+\beta\kappa)\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right) = 2 + \left(\frac{e}{2b-e}\right)\beta\kappa - \left(\frac{b-e}{2b-e}\right)\beta^2\kappa^2.$$

This product is a negative quadratic that is maximized at $\kappa = -e/(2\beta(2b-e))$. If this quantity is greater than 3, the above equation is positive and increasing for all relevant κ. Since $\partial\widetilde{\text{LHS}}/\partial\kappa$ is negative and decreasing (increasing in magnitude), this implies that $\partial^2\text{LHS}/\partial\kappa < 0$ and LHS is concave. For $-e/(2\beta(2b-e)) \geq 3$ we require

$$-\frac{e}{2\beta\,(2b-e)} \geq 3 \quad\Longleftrightarrow\quad -\frac{1}{2}\left(\frac{2b+e}{e}\right) \geq 3 \quad\Longleftrightarrow\quad e \leq -\frac{2}{7}b \quad\Longleftrightarrow\quad r_{eb} \leq -\frac{2}{7}.$$

By definition, there is a decreasing portion of LHS if and only if $\kappa^{\perp} < 3$. Then we need to show that $\kappa^{\perp} < 3$ implies $r_{eb} \leq -2/7$. Since we have shown that $\kappa^{\perp}$ is increasing in r_{eb}, it is sufficient to show that $r_{eb} = -2/7$ implies $\kappa^{\perp} \geq 3$. To see this, substitute into equation (7) with $r_{eb} = -2/7$,

$$\left.\frac{\partial\widetilde{\text{LHS}}}{\partial\kappa}\right|_{r_{eb}=-\frac{2}{7}} = 2 + \left(3 - 6\left(\frac{\frac{9}{7}}{\frac{16}{7}}\right)\right)\beta\kappa - 5\left(\frac{\frac{9}{7}}{\frac{16}{7}}\right)\beta^2\kappa^2 \propto 32 - 6\beta\kappa - 45\beta^2\kappa^2.$$

Solving this quadratic in $\beta\kappa$ yields

$$\beta\kappa = -\frac{1}{90}\left(6+\sqrt{36+4\,(45)\,(32)}\right) = -\frac{1}{15}\left(1 \pm \sqrt{161}\right).$$

Since $\beta\kappa \geq 0$, only the "minus" solution is negative. This yields

$$\begin{aligned}\kappa = \frac{1}{15}\left(\sqrt{161}-1\right)\left(\frac{1}{\beta}\right) &= \frac{1}{15}\left(\sqrt{161}-1\right)\left(\frac{4-r_{eb}^2}{r_{eb}^2}\right)\\ &= \frac{1}{15}\left(\sqrt{161}-1\right)\left(\frac{4-\left(-\frac{2}{7}\right)^2}{\left(-\frac{2}{7}\right)^2}\right) = \frac{16}{5}\left(\sqrt{161}-1\right) \approx 37.403.\end{aligned}$$

Then $r_{eb} = -2/7$ implies $\kappa^{\perp} > 3$, hence $\kappa^{\perp} \leq 3$ implies $r_{eb} < -2/7$. Then LHS is concave when it is decreasing.

We now show that RHS is convex for $\kappa \in [3/2, 3]$, thus when LHS is possibly decreasing RHS is convex. We show this directly by examining the second derivative and showing that it is positive.

$$\frac{\partial \text{RHS}}{\partial \kappa} \propto -2\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa \left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)\left(2-\left(1-\beta\right)\kappa\right)$$
$$-\left(1-\beta\right)\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)^2;$$

$$\begin{aligned}
\frac{\partial^2 \text{RHS}}{\partial \kappa^2} &\propto -2\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2 \left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)\left(2-\left(1-\beta\right)\kappa\right) \\
&\quad + 2\left(1-\bar{\tau}_{s,\rho}\right)^2 b^2\beta^4\kappa^2\left(2-\left(1-\beta\right)\kappa\right) \\
&\quad + 2\left(1-\beta\right)\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right) \\
&\quad + 2\left(1-\beta\right)\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right) \\
&\propto -\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)\left(2-\left(1-\beta\right)\kappa\right) \\
&\quad + b\beta^2\kappa^2\left(2-\left(1-\beta\right)\kappa\right)+2\left(1-\beta\right)\kappa\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right) \\
&= \left(3\left(1-\beta\right)\kappa-2\right)\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)+b\beta^2\kappa^2\left(2-\left(1-\beta\right)\kappa\right).
\end{aligned}$$

By definition $1-\beta \geq 2/3$, and by assumption $\kappa > 3/2$. Then $3(1-\beta)\kappa > 3$. It follows that

$$\frac{\partial^2 \text{RHS}}{\partial \kappa^2} \geq \left(\left(\left(2b-\bar{\tau}_{s,\rho}e\right)-\frac{1}{2}\left(1-\bar{\tau}_{s,\rho}\right) b\beta^2\kappa^2\right)+b\beta^2\kappa^2\left(2-\left(1-\beta\right)\kappa\right)\right)\sigma_\theta^2\bar{\tau}_{s,\theta} \geq 0.$$

Moreover, this inequality is strict so long as $\sigma_\theta^2\bar{\tau}_{s,\theta} > 0$. Then RHS is convex for $\kappa > 3/2$.

Putting these arguments together gives the following. LHS and RHS intersect on the interval $\kappa \in [0, 3]$. RHS is decreasing, and if LHS is increasing they have a unique intersection. If LHS is not increasing, it is increasing for small values of κ and decreasing for large values of κ, and where it is decreasing it is concave. For any possible values on which LHS is decreasing, RHS is convex. Then there is at most one intersection on the region where LHS is decreasing. If LHS is greater than RHS at the point at which it becomes decreasing, the curves do not intersect at any point to the right because LHS$(\bar{\kappa})$ > RHS$(\bar{\kappa}) = 0$. Then the curves intersect at a unique point, and equilibrium κ is unique. This directly implies that equilibrium price strategies are unique.

Proof of Proposition 2

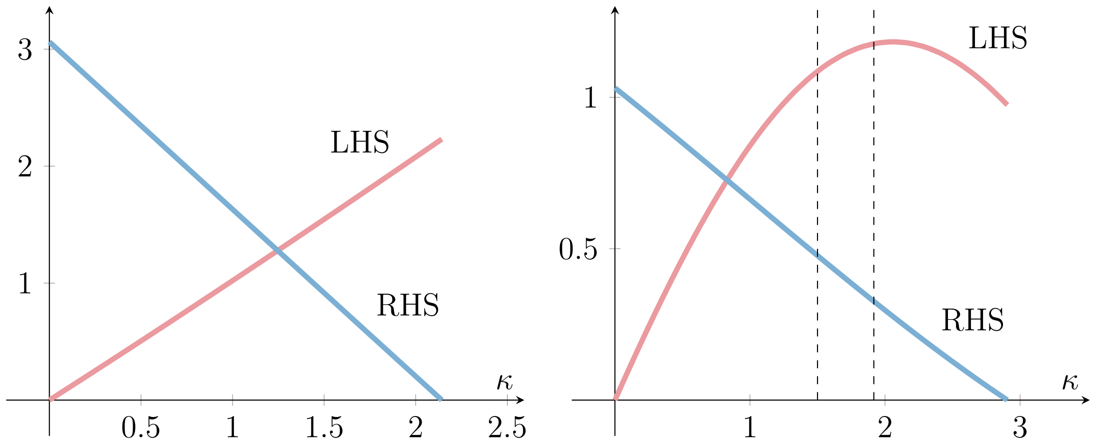

Figure 1: A graphical depiction of the proof of equilibrium existence and uniqueness. The existence of an equilibrium amounts to finding a κ such that $\text{LHS}(\kappa) = \text{RHS}(\kappa)$. Since $\text{LHS}(0) < \text{RHS}(0)$ and $\text{LHS}(\bar{\kappa}) > \text{RHS}(\bar{\kappa})$ and both functions are continuous, such a κ is guaranteed to exist. Additionally, RHS is decreasing. We show that either LHS is increasing (left panel) or increasing and then decreasing (right panel). In the former case, it is clear that there is a unique point of intersection and hence a unique equilibrium. In the latter case, we show that LHS is concave where it is decreasing and RHS is convex anywhere LHS is decreasing. Then LHS $-$ RHS is concave, ensuring that equilibrium κ is unique.

From Proposition 1 we know that the values of p_θ and p_ρ in equilibrium are

$$p_\rho = \frac{1 - \beta\kappa\left(\frac{b-e}{2b-e}\right)}{2 - \frac{e}{b}\bar{\tau}_{s,\rho} - \frac{1}{2}\beta^2\kappa^2(1-\bar{\tau}_{s,\rho})} \text{ and } p_\theta = \frac{1}{2+\beta\kappa}.$$

If we multiply the top and bottom of the expression for p_θ by $1 - \beta\kappa\left(\frac{b-e}{2b-e}\right)$, then the numerators are the same and we only need to compare the denominators of each expression. The denominator of p_θ becomes

$$(2+\beta\kappa)\left(1 - \beta\kappa\left(\frac{b-e}{2b-e}\right)\right) = 2 - \left(\frac{b-e}{2b-e}\right)\beta^2\kappa^2 - 2\left(\frac{b-e}{2b-e}\right)\beta\kappa + \beta\kappa$$

When $0 < e \leq b$, then $\left(\frac{b-e}{2b-e}\right) \in \left[0, \frac{1}{2}\right)$, and

$$\begin{aligned} 2 - \left(\frac{b-e}{2b-e}\right)\beta^2\kappa^2 - 2\left(\frac{b-e}{2b-e}\right)\beta\kappa + \beta\kappa &\geq 2 - \frac{1}{2}\beta^2\kappa^2(1-\bar{\tau}_{s,\rho}) - \frac{1}{2}\beta^2\kappa^2\bar{\tau}_{s,\rho} \\ &> 2 - \frac{e}{b}\bar{\tau}_{s,\rho} - \frac{1}{2}\beta^2\kappa^2(1-\bar{\tau}_{s,\rho}). \end{aligned}$$

The last inequality follows from the fact that

$$\frac{1}{2}\beta^2\kappa^2 \leq \frac{1}{2}\beta\kappa = \frac{e}{b}\frac{be}{2(4b^2-e^2)}\kappa < \frac{e}{b}.$$

Since the denominator of p_θ is at least as large as that of p_ρ, then $p_\rho < p_\theta$.

When $-b \leq e < 0$, then $\left(\frac{b-e}{2b-e}\right) \in \left[\frac{1}{2}, \frac{2}{3}\right)$, and $\frac{e}{b} \leq \frac{1}{2}\beta^2\kappa^2$. Therefore the above inequalities flip and for these range of parameters, $p_\rho \leq p_\theta$.

Proof of Proposition 3

Effect of $\bar{\tau}_{s,\theta}$

From Proposition 1 we know that there is a unique $\kappa \in \left[0, \frac{2}{1-\beta}\right]$ such that the LHS and the RHS of equation (6) are equal. Moreover, for $\kappa = 0$ the LHS is 0 and the right hand side is positive, and for $\kappa = \frac{2}{1-\beta}$ the RHS is 0 and the LHS is positive. Therefore, for all κ larger than the κ which satisfies the equation, the LHS is larger than the RHS.

As $\bar{\tau}_{s_\theta}$ increases, the LHS of the equation is constant while the RHS increases for all κ. This shift up of the RHS will increase the κ which satisfies the equation. Because of the inverse relationship between κ and p_θ, p_θ will decrease as as $\bar{\tau}_{s,\theta}$ increases.

Effect of $\bar{\tau}_{s,\rho}$

From our definition of $\bar{\tau}_{s,x}$, we have

$$\bar{\tau}_{s,x} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_{s,x}^2} \quad \implies \quad \sigma_{s,x}^2 = \left(\frac{1-\bar{\tau}_{s,x}}{\bar{\tau}_{s,x}}\right)\sigma_x^2.$$

Then in the fixed point equation,

$$\begin{aligned}\mathrm{LHS}\left(\kappa\right) &= (2+\beta\kappa)^2\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)^2\left(1-\bar{\tau}_{s,\rho}\right)\bar{\tau}_{s,\rho}\sigma_\rho^2\kappa, \text{ and}\\ \frac{\partial \mathrm{LHS}}{\partial\bar{\tau}_{s,\rho}}\left(\kappa\right) &= (2+\beta\kappa)^2\left(1-\beta\kappa\left(\frac{b-e}{2b-e}\right)\right)^2\left(1-2\bar{\tau}_{s,\rho}\right)\sigma_\rho^2\kappa.\end{aligned}$$

The first equality follows directly from the definition of LHS and the above equation for $\sigma_{s,\rho}^2$. In particular, $\sigma_{s,\rho}^2\bar{\tau}_{s,\rho}^2 = \left(1-\bar{\tau}_{s,\rho}\right)\bar{\tau}_{s,\rho}\sigma_\rho^2$. The second equality is then immediate. This implies that $\partial\mathrm{LHS}/\partial\bar{\tau}_{s,\rho}$ is linear and decreasing in $\bar{\tau}_{s,\rho}$, and is positive when $\bar{\tau}_{s,\rho} < 1/2$ and negative when $\bar{\tau}_{s,\rho} > 1/2$.

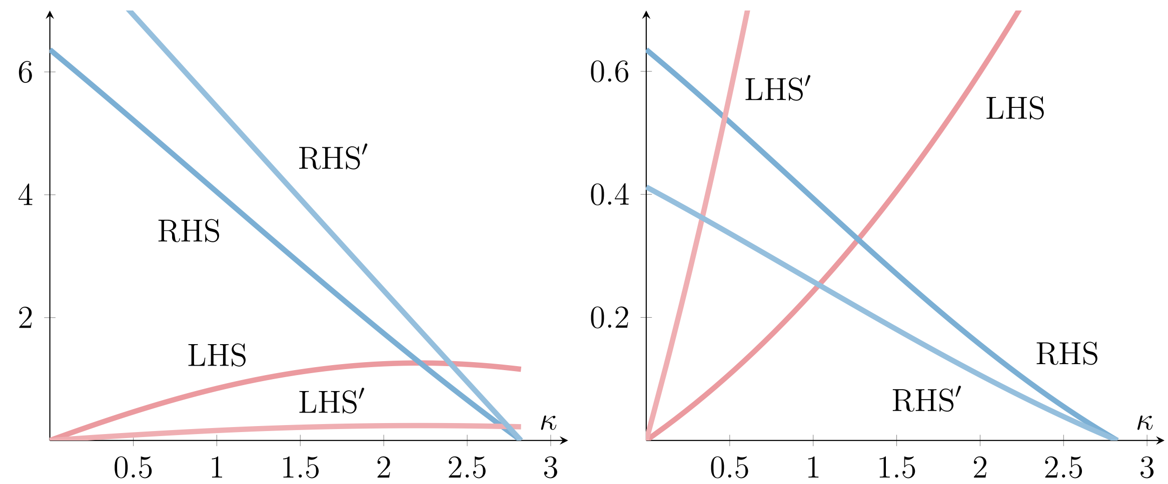

Figure 2: A graphical depiction of the response of κ to $\bar{\tau}_{s,\rho}$. When $\bar{\tau}_{s,\rho} \geq 1/2$ and $e < 0$, an increase in $\bar{\tau}_{s,\rho}$ increases RHS and decreases LHS, pushing κ rightward (left panel). When $\bar{\tau}_{s,\rho} \leq 1/2$ and $e > 0$, an increase in $\bar{\tau}_{s,\rho}$ decreases RHS and increases LHS, pushing κ leftward (right panel).

Additionally in the fixed point equation,

$$\mathrm{RHS}(\kappa) = \left(2b - \frac{1}{2}b\beta^2\kappa^2 + \left(\frac{1}{2}b\beta^2\kappa^2 - e\right)\bar{\tau}_{s,\rho}\right)^2 (2 - (1-\beta)\kappa)\,\sigma_\theta^2 \bar{\tau}_{s,\theta}, \text{ and}$$

$$\frac{\partial \mathrm{RHS}}{\partial \bar{\tau}_{s,\rho}}(\kappa) = 2\left(\frac{1}{2}b\beta^2\kappa^2 - e\right)\left(2b - \frac{1}{2}b\beta^2\kappa^2 + \left(\frac{1}{2}b\beta^2\kappa^2 - e\right)\bar{\tau}_{s,\rho}\right)(2 - (1-\beta)\kappa)\,\sigma_\theta^2 \bar{\tau}_{s,\theta}.$$

Therefore $\partial \mathrm{RHS}/\partial \bar{\tau}_{s,\rho}$ is linear and increasing in $\bar{\tau}_{s,\rho}$, and is positive when $e < 0$ and negative when $e > 0$. This follows directly from rearrangement of the internal term,

$$\frac{1}{2}b\beta^2\kappa^2 - e = \left(\frac{1}{2}(\beta\kappa)\left(\frac{be}{4b^2 - e^2}\kappa\right) - 1\right)e.$$

If $e < 0$, $be/(4b^2 - e^2) < 0$; if $e > 0$, $\beta\kappa < 1$ and $be\kappa/(4b^2 - e^2) < 1$, so difference in parentheses is negative. In either case, the term is signed as $-e$. Then when $e < 0$ this term is positive and when $e > 0$ this term is negative. Lemma 11 follows directly from the above observations.

Lemma 11. *When $\bar{\tau}_{s,\rho} \geq 1/2$ and $e < 0$, κ is increasing and p_θ is decreasing in $\bar{\tau}_{s,\rho}$. When $\bar{\tau}_{s,\rho} \leq 1/2$ and $e > 0$, κ is decreasing and p_θ is increasing in $\bar{\tau}_{s,\rho}$.*

Lemma 12 (Limiting cases of common information)**.** *When $\bar{\tau}_{s,\rho} \in \{0, 1\}$,*

$$\kappa = \frac{1}{p_\theta} \implies p_\theta = \frac{1-\beta}{2}.$$

Proof. The second equality follows directly from the first. Recall that $\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 = (1-\bar{\tau}_{s,\rho})\,\bar{\tau}_{s,\rho}\sigma_\rho^2$. Since all endogenous terms are bounded, when $\bar{\tau}_{s,\rho} \in \{0,1\}$ we have

$$\kappa = \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 p_\rho^2 + \left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right) \bar{\tau}_{s,\theta}^2 p_\theta^2} = \frac{\sigma_{s,\rho}^2 \bar{\tau}_{s,\theta} p_\theta}{\left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right) \bar{\tau}_{s,\theta}^2 p_\theta^2} = \frac{1}{p_\theta}.$$

□

Lemma 13 (Effect of $\bar{\tau}_{s,\rho}$ on p_θ (first hard case))**.** *When $e > 0$, there is a $\tau^\star > 1/2$ such that whenever $\bar{\tau}_{s,\rho} > \tau^\star$, κ is increasing and p_θ is decreasing in $\bar{\tau}_{s,\rho}$, and whenever $\bar{\tau}_{s,\rho} < \tau^\star$, κ is decreasing and p_θ is increasing in $\bar{\tau}_{s,\rho}$.*

Proof. This is a loose proof.

We have already established that if such a $\tau^\star$ exists, it is greater than $1/2$. Since $p_\theta = (1-\beta)/2$ when $\bar{\tau}_{s,\rho} \in \{0,1\}$ and κ is continuous in the parameters of the problem, there is a value $\hat{\tau}^\star$ such that $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} = \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ at $\hat{\tau}^\star$. Because both of these partial derivatives are linear, a non-marginal increase in $\bar{\tau}_{s,\rho}$ leads to a point where $\partial\text{RHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{LHS}/\partial\bar{\tau}_{s,\rho}$; as discussed earlier, a non-marginal increase in $\bar{\tau}_{s,\rho}$ from this point leads (naïvely) to an increase in κ.

We can additionally show that when $e > 0$ and $\bar{\tau}_{s,\rho} > 1/2$, $\partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ is increasing and $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho}$ is decreasing in κ. This add-on effect implies that $\partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ is even larger and $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho}$ is even smaller than we would naïvely expect, pushing κ even higher. Then all effects point in the same direction. Letting $\tau^\star = \hat{\tau}^\star$ completes the proof. □

Lemma 14 (Effect of $\bar{\tau}_{s,\rho}$ on p_θ (first hard case))**.** *When $e < 0$, there is a $\tau^\star \leq 1/2$ such that whenever $\bar{\tau}_{s,\rho} < \tau^\star$, κ is deceasing and p_θ is increasing in $\bar{\tau}_{s,\rho}$, and whenever $\bar{\tau}_{s,\rho} > \tau^\star$, κ is increasing and p_θ is decreasing in $\bar{\tau}_{s,\rho}$.*

Proof. This is a loose proof.

We have already established that if such a $\tau^\star$ exists, it is less than $1/2$. As noted earlier, $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho}$ is positive and decreasing in $\bar{\tau}_{s,\rho}$ and $\partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ is positive and increasing in $\bar{\tau}_{s,\rho}$. If $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$, then κ is locally decreasing in $\bar{\tau}_{s,\rho}$.

From earlier results, we have

$$\frac{\partial\text{LHS}}{\partial\bar{\tau}_{s,\rho}}(\kappa) = \left(\frac{1-2\bar{\tau}_{s,\rho}}{(1-\bar{\tau}_{s,\rho})\,\bar{\tau}_{s,\rho}}\right)\text{LHS}(\kappa)\,, \text{ and}$$
$$\frac{\partial\text{RHS}}{\partial\bar{\tau}_{s,\rho}}(\kappa) = \left(\frac{b\beta^2\kappa^2 - 2e}{2b - \frac{1}{2}b\beta^2\kappa^2 + \left(\frac{1}{2}b\beta^2\kappa^2 - e\right)\bar{\tau}_{s,\rho}}\right)\text{RHS}(\kappa)\,.$$

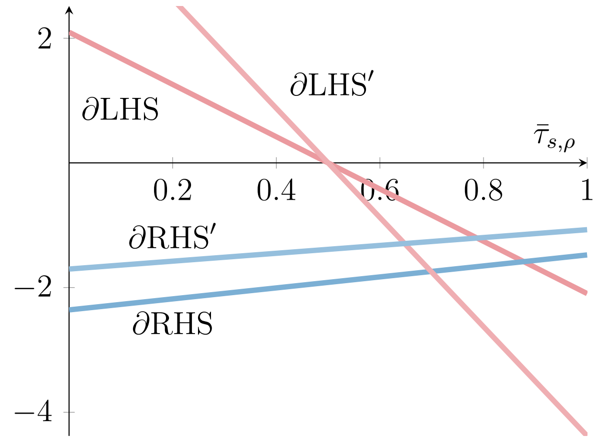

Figure 3: A graphical depiction of the comparative statics of κ with respect to $\bar{\tau}_{s,\rho}$ when $e > 0$ and $\bar{\tau}_{s,\rho} > 1/2$. In this case, both LHS and RHS decrease. We show that for sufficiently high $\bar{\tau}_{s,\rho}$, LHS decreases at a faster rate than RHS. The existence of a unique threshold determining which term decreases faster follows from the observation that from the point at which they decrease equally (the $\bar{\tau}_{s,\rho}$ at which $\partial\kappa/\partial\bar{\tau}_{s,\rho} = 0$), a slight increase in $\bar{\tau}_{s,\rho}$ moves into a region where LHS is decreasing faster than RHS. While the first-order effect is zero, the second-order effect implies that κ is increasing. We show that in this case, $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho}$ is decreasing in κ while $\partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ is increasing in κ, so the additional second-order effect (due to the derivatives shifting) points in the same direction.

In any equilibrium, $\text{LHS}(\kappa) = \text{RHS}(\kappa)$, so checking $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ is equivalent to checking

$$
\begin{aligned}
& \frac{1-2\bar{\tau}_{s,\rho}}{(1-\bar{\tau}_{s,\rho})\,\bar{\tau}_{s,\rho}} > \frac{\beta^2\kappa^2 - 2r_{eb}}{2 - \frac{1}{2}\beta^2\kappa^2 + \left(\frac{1}{2}\beta^2\kappa^2 - r_{eb}\right)\bar{\tau}_{s,\rho}} \\
\iff\quad & \left(\frac{2 - r_{eb}\bar{\tau}_{s,\rho}}{1-\bar{\tau}_{s,\rho}} - \frac{1}{2}\beta^2\kappa^2\right)(1-2\bar{\tau}_{s,\rho}) > \left(\beta^2\kappa^2 - 2r_{eb}\right)\bar{\tau}_{s,\rho} \\
\iff\quad & (2 - r_{eb}\bar{\tau}_{s,\rho})(1-2\bar{\tau}_{s,\rho}) > \left(\frac{1}{2}\beta^2\kappa^2 - 2r_{eb}\bar{\tau}_{s,\rho}\right)(1-\bar{\tau}_{s,\rho}) \\
\iff\quad & 2(1-2\bar{\tau}_{s,\rho}) > \frac{1}{2}(1-\bar{\tau}_{s,\rho})\,\beta^2\kappa^2 - \bar{\tau}_{s,\rho} r_{eb} \\
\iff\quad & 2 - \frac{1}{2}\beta^2\kappa^2 > \left(4 - r_{eb} - \frac{1}{2}\beta^2\kappa^2\right)\bar{\tau}_{s,\rho}. \qquad (10)
\end{aligned}
$$

Applying bounds to inequality (10) gives that $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ when $\bar{\tau}_{s,\rho} < 3/10$, regardless of the other parameters in the model. Then when $e < 0$, κ is decreasing in $\bar{\tau}_{s,\rho}$ when $\bar{\tau}_{s,\rho} < 3/10$ and increasing when $\bar{\tau}_{s,\rho} > 1/2$. We now address the remaining gap, $\bar{\tau}_{s,\rho} \in [3/10, 1/2]$.

Note that inequality (10) becomes more difficult to satisfy as κ increases. In particular, the derivative of the left-hand side is $-\beta^2\kappa$ while the derivative of the right-hand side is

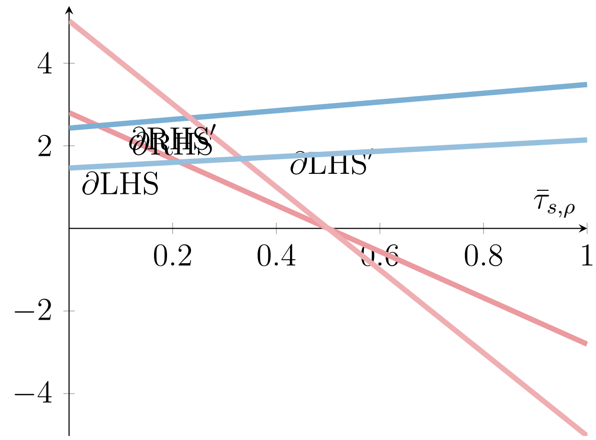

Figure 4: A graphical depiction of the comparative statics of κ with respect to $\bar{\tau}_{s,\rho}$ when $e < 0$ and $\bar{\tau}_{s,\rho} < 1/2$. This is just a doodle for intuition.

$-\beta^2\kappa\bar{\tau}_{s,\rho}$; since $\bar{\tau}_{s,\rho} < 1/2$, the derivative of the right-hand side is less negative (hence larger) than the derivative of the left-hand side. By earlier arguments, we know that there is a $\hat{\tau}^\star$ such that $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} = \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$. Consider the effect on κ of a small *decrease* in $\bar{\tau}_{s,\rho}$ from this point. Since $\bar{\tau}_{s,\rho}$ is moving into the range on which $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$, κ should intuitively increase (this direction is "backwards", since we are looking at a decrease in $\bar{\tau}_{s,\rho}$, and thus a negative derivative), ignoring fixed point effects. However, since $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} = \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ at $\hat{\tau}^\star$, fixed point effects are relevant. If κ were to decrease following this small decrease in $\bar{\tau}_{s,\rho}$, inequality (10) becomes easier to satisfy. Then $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} > \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$ in both first- and second-order effects, implying that κ increases. This contradicts the assumption that κ decreases. Then a small decrease in $\bar{\tau}_{s,\rho}$ results in an increase in κ.

It then follows that from any $\bar{\tau}_{s,\rho}$ such that $\partial\text{LHS}/\partial\bar{\tau}_{s,\rho} = \partial\text{RHS}/\partial\bar{\tau}_{s,\rho}$, κ is *increasing* to the left of this intersection (we showed κ was decreasing with a decrease in $\bar{\tau}_{s,\rho}$, therefore κ is increasing in $\bar{\tau}_{s,\rho}$). Letting $\tau^\star$ be the maximum such $\hat{\tau}^\star$ completes the proof. $\square$

Proof of Proposition 4

From Lemma 5, equilibrium first period prices must satisfy

$$p^c_{i,1} = \frac{1}{2b}\mathbb{E}\left[bc_i + a + ep^c_{j,1} + \frac{1}{2b}\left(a - bc_i + e\mathbb{E}\left[p^c_{j,2}\middle|\,\rho, s_\rho, p_1\right]\right)\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p^\star_{j,2}\middle|\,\rho, s_\rho, p_1\right]\middle|\, s_\rho, s_{i,\theta}\right].$$

The expectation of second period price is

$$\mathbb{E}\left[\mathbb{E}\left[p^c_{j,2}\middle|\,\rho, s_\rho, p_1\right]\middle|\, s_{i,\theta}, s_\rho\right] = \frac{a}{2b-e} + \frac{\beta-1}{2}\mathbb{E}\left[\mathbb{E}[c_i|\rho, s_\rho, p_{i,1}]\middle|\, s_{i,\theta}, s_\rho\right] + \frac{be}{4b^2-e^2}\mathbb{E}\left[\mathbb{E}[c_j|\rho, s_\rho, p_{i,1}]\middle|\, s_{i,\theta}, s_\rho\right].$$

The conditional expectation of each firm's cost in the second period is

$$\mathbb{E}[c_k|\rho, s_\rho, p_{k,1}] = \rho + \mathbb{E}[\theta_k|s_{k,\theta}].$$

The expectation of this conditional expectation in the first period are

$$\begin{aligned}\mathbb{E}\left[\mathbb{E}[c_i|\rho, s_\rho, p_{i,1}]\right| s_{i,\theta}, s_\rho] &= \mathbb{E}[\rho|s_\rho] + \mathbb{E}[\theta_i|s_{i,\theta}] \\ \mathbb{E}\left[\mathbb{E}[c_j|\rho, s_\rho, p_{i,1}]\right| s_{i,\theta}, s_\rho] &= \mathbb{E}[\rho|s_\rho] + \mu_\theta\end{aligned}$$

Additionally, an increase in first period price effects the public expectation of cost in the second period directly by the coefficient on the signal.

$$\frac{\partial \mathbb{E}[c_i|\rho, s_\rho, p_{i,1}]}{\partial p_{i,1}} = \frac{\partial E[c_i|s_{i,\theta}]}{\partial s_{i,\theta}} \frac{\partial s_{i,\theta}}{\partial p_{i,1}} = \frac{\bar{\tau}_{i,\theta}}{\tilde{p}_\theta} = \frac{1}{p_\theta}$$

Given the simplified information structure, the first order condition in becomes

$$\begin{aligned}2bp^c_{i,1} &= b(\mathbb{E}[\theta_i|s_{i,\theta}] + \mathbb{E}[\rho|s_\rho]) + a + e\mathbb{E}[p^c_{j,1}|s_\rho] \\ &\quad + b\beta\frac{1}{p_{\theta,c}}\left[\frac{a}{2b-e} + \frac{\beta-1}{2}(\mathbb{E}[\theta_i|s_{i,\theta}] + E[\rho|s_\rho]) + \frac{be}{4b^2-e^2}\left(\mathbb{E}[\rho|s_\rho] + \mu_\theta\right)\right]\end{aligned}$$

Rearranging and matching coefficients, we have that any linear equilibrium satisfies the following system of equations.

$$\begin{aligned}p_{\theta,c}(1-2p_{\theta,c}) &= \frac{\beta(1-\beta)}{2} \\ p_{\theta,c}\left(1 + \frac{e}{b}p_{\rho,c} - 2p_{\rho,c}\right) &= \left(\frac{1-\beta}{2} - \frac{be}{4b^2-e^2}\right)\beta \\ p_{\theta,c}\left(\frac{a}{b} + \frac{e}{b}(p_{0,c} + p_{\theta,c}\mu_\theta) - 2p_{0,c}\right) &= -\beta\left(\frac{a}{2b-e} + \frac{be\mu_\theta}{4b^2-e^2}\right)\end{aligned}$$

where $p^c_{i,1} = p_{0,c} + p_{\rho,c}E[\rho|s_\rho] + p_{\theta,c}E[\theta_i|s_{i,\theta}]$. The first equation has two solutions for $p_{\theta,c}$, but the second order condition only holds for $p_{\theta,c} = (1-\beta)/2$. Solving the remaining equations given this value for p_θ we get

$$
\begin{aligned}
p_{\rho,c}(2b-e) &= b + \frac{2\beta b}{1-\beta}\left(\frac{\beta-1}{2} + \frac{be}{4b^2-e^2}\right) \\
\Rightarrow p_{\rho,c} &= \frac{b}{2b-e} + \frac{be}{4b^2-e^2}\beta \\
p_{0,c}(2b-e) &= a + \frac{(1-\beta)e\mu_\theta}{2} + \frac{2b\beta}{1-\beta}\left(\frac{a}{2b-e} + \frac{be}{4b^2-e^2}\mu_\theta\right) \\
\Rightarrow p_{0,c} &= \frac{2a+(1-\beta)e\mu_\theta}{2(2b-e)} + \frac{2b\beta}{1-\beta}\left(\frac{a}{2b-e} + \frac{be}{4b^2-e^2}\mu_\theta\right)
\end{aligned}
$$

Because $\kappa \leq 2/(1-\beta)$ with the inequality strict for $e \neq 0$, then

$$
p_\theta = \frac{1}{2+\beta\kappa} \geq \frac{1}{2+\frac{2\beta}{1-\beta}} = \frac{1-\beta}{2} = p_{\theta,c}
$$

with the inequality strict when $e \neq 0$.

Defining $r = \frac{e}{b}$ we know that $r \in [-1,1]$ and the sign of r is the same as the sign of e. Then we can write both $p_{\rho,c}$ and p_ρ in terms of r,

$$
p_{\rho,c} = \frac{1}{2-r} + \frac{r}{4-r^2}\beta \text{ and } p_\rho = \frac{\frac{(2-r)-(1-r)\beta\kappa}{2-r}}{2-r\bar{\tau}_{s,\rho} - \frac{1}{2}\beta^2\kappa^2(1-\bar{\tau}_{s,\rho})}.
$$

Becuase $\beta\kappa \leq \frac{e}{b} \leq 1$ then p_ρ can be bounded as follows when $r \geq 0$

$$
p_\rho \leq \frac{(2-r)-(1-r)\beta\kappa}{(2-r)^2} \leq \frac{1}{2-r} \leq p_{\rho,c}.
$$

Proof of Proposition 5

Comparing the first order conditions in Lemma 3 and Lemma 5, all terms are the same except $\mathbb{E}[\frac{\partial}{\partial p_{i,1}}\mathbb{E}\left[p_{j,2}\middle|\,\rho, p_{i,1}, s\right]|s]$ and $\mathbb{E}[\mathbb{E}\left[p_{j,2}\middle|\,\rho, p_{i,1}, s\right]|s]$. From Proposition 4 we know that the first term is constant for all signals, and is larger when firms share common cost information than when they do not. Taking the expectation of the second term it is the same in either case.

$$
\begin{aligned}
&\mathbb{E}[\mathbb{E}[\mathbb{E}\left[p_{j,2}\middle|\,\rho, p_{i,1}, s\right]|s]] \\
&= \frac{1}{4b^2-e^2}\left(\mathbb{E}[(2b+e)a + 2b^2(\mu_\theta + E[\rho|s_{i,\rho}])] + be\mathbb{E}[\kappa(\mathbb{E}[p_{i,1}|s] - p_0 + p_\theta\mu_\theta + p_\rho\mathbb{E}[\rho|s])]\right)
\end{aligned}
$$

The later term equals zero in both the sharing and non-sharing settings as $\mathbb{E}[\mathbb{E}[p_{i,1}|s] -$

$p_0+p_\theta\mu_\theta+p_\rho E[\rho|s])] = 0$. Moreover, the former term only consists of constants and expected costs which are the same in each case. Therefore

$$\mathbb{E}[p_{i,2}] = \mathbb{E}[p_{i,2}^c] = \frac{1}{4b^2-e^2}\left((2b+e)+2b^2(\mu_\theta+\mu_\rho)\right)$$

Imposing symmetry and taking the expectation of optimal pricing equation we have,

$$\mathbb{E}[2bp_{i,1}] = \mathbb{E}\left[\mathbb{E}\left[bc_i + a + ep_{i,1} + \frac{1}{2b}\left(a - bc_i + e\mathbb{E}\left[p_{j,2}^c \middle| \rho, s_\rho, p_1\right]\right)\frac{be}{4b^2-e^2}\kappa \middle| s_\rho, s_{i,\theta}\right]\right].$$

where all terms are the same except κ, which is larger and denoted by κ^c in the case where firms share common cost information. Then

$$\begin{aligned}
\mathbb{E}[p_{i,1}] &= \frac{1}{2b-e}\left(a + b(\mu_\rho+\mu_\theta) + \frac{e\kappa}{2(4b^2-e^2)}\mathbb{E}\left[\mathbb{E}\left[\left(a - bc_i + e\mathbb{E}\left[p_{j,2}\middle| \rho, s_\rho, p_1\right]\right)\middle| s_\rho, s_{i,\theta}\right]\right]\right) \\
&\le \frac{1}{2b-e}\left(a + b(\mu_\rho+\mu_\theta) + \frac{e\kappa^c}{2(4b^2-e^2)}\mathbb{E}\left[\mathbb{E}\left[\left(a - bc_i + e\mathbb{E}\left[p_{j,2}^c\middle| \rho, s_\rho, p_1\right]\right)\middle| s_\rho, s_{i,\theta}\right]\right]\right) \\
&= \mathbb{E}[p_{i,1}^c]
\end{aligned}$$

Proof of Lemma 6

Ex ante expected profits in the first period are given by

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[\pi_{i,1}\middle| s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(a - bp_i + ep_j\right)\left(p_i - c_i\right)\middle| s_i\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[ap_i - ac_i - bp_i^2 + bc_ip_i + ep_ip_j - ec_ip_j\middle| s_i\right]\right].
\end{aligned}$$

Any terms which contain only first powers of uncorrelated variables are invariant to the investment in precision, and can be subsumed into an additional parameter. For example,

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[ap_i\middle| s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[ap_{i0} + ap_{i\theta}\mathbb{E}\left[\theta_i\middle| s_{i,\theta}\right] + ap_{i\rho}\mathbb{E}\left[\rho\middle| s_{i,\rho}\right]\middle| s_i\right]\right] \\
&= ap_{i0} + ap_{i\theta}\mathbb{E}\left[\mathbb{E}\left[\theta_i\middle| s_i\right]\right] + ap_{i\rho}\mathbb{E}\left[\mathbb{E}\left[\rho\middle| s_i\right]\right] \\
&= ap_{i0} + ap_{i\theta}\mathbb{E}\left[\theta_i\right] + ap_{i\rho}\mathbb{E}\left[\rho\right] = ap_{i0} + ap_{i\theta}\mu_\theta + ap_{i\rho}\mu_\rho; \\
\mathbb{E}\left[\mathbb{E}\left[ac_i\middle| s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(\theta_i + \rho\right)a\middle| s_i\right]\right] \\
&= a\mathbb{E}\left[\mathbb{E}\left[\theta_i + \rho\middle| s_i\right]\right] = a\theta_i + a\rho.
\end{aligned}$$

Lemma 15. *The interrelations of the conditional expected values of cost components are as* *follows:*

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[\theta_i | s_i\right]^2\right] &= \frac{\tau_{i,\theta}}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta} + \mu_\theta^2, \\
\mathbb{E}\left[\mathbb{E}\left[\rho | s_i\right]^2\right] &= \frac{\tau_{i,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho} + \mu_\rho^2, \\
\mathbb{E}\left[\mathbb{E}\left[\rho | s_i\right]\mathbb{E}\left[\rho | s_j\right]\right] &= \frac{\tau_{i,\rho}\tau_{j,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho} + \mu_\rho^2.
\end{aligned}$$

Then first-period profits rely on the investment in precision only through the last four parameters.

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[p_i^2 \middle| s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(p_{i0}+p_{i\theta}\mathbb{E}\left[\theta_i | s_i\right]+p_{i\rho}\mathbb{E}\left[\rho | s_i\right]\right)^2 \middle| s_i\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[p_{i0}^2+2p_{i0}p_{i\theta}\mathbb{E}\left[\theta_i | s_i\right]+2p_{i0}p_{i\rho}\mathbb{E}\left[\rho | s_i\right] \middle| s_i\right]\right] \\
&\quad + \mathbb{E}\left[\mathbb{E}\left[p_{i\theta}^2\mathbb{E}\left[\theta_i | s_i\right]^2+2p_{i\theta}p_{i\rho}\mathbb{E}\left[\theta_i | s_i\right]\mathbb{E}\left[\rho | s_i\right]+p_{i\rho}^2\mathbb{E}\left[\rho | s_i\right]^2 \middle| s_i\right]\right] \\
&= \frac{\tau_{i,\theta}p_{i\theta}^2}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta} + \frac{\tau_{i,\rho}p_{i\rho}^2}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho} + C_{i1}.
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[c_ip_i | s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(\theta_i+\rho\right)\left(p_{i0}+p_{i\theta}\mathbb{E}\left[\theta_i | s_i\right]+p_{i\rho}\mathbb{E}\left[\rho | s_i\right]\right) | s_i\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[p_{i\theta}\theta_i\mathbb{E}\left[\theta_i | s_i\right]+p_{i\rho}\rho\mathbb{E}\left[\rho | s_i\right] | s_i\right]\right] + C_{i21} \\
&= \mathbb{E}\left[p_{i\theta}\mathbb{E}\left[\theta_i | s_i\right]^2+p_{i\rho}\mathbb{E}\left[\rho | s_i\right]^2\right] + C_{i21} \\
&= \frac{\tau_{i,\theta}p_{i\theta}}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta} + \frac{\tau_{i,\rho}p_{i\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho} + C_{i2}.
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[p_ip_j | s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[p_ip_j | s_i, s_j\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(p_{i0}+p_{i\theta}\mathbb{E}\left[\theta_i | s_i\right]+p_{i\rho}\mathbb{E}\left[\rho | s_i\right]\right)\left(p_{j0}+p_{j\theta}\mathbb{E}\left[\theta_j | s_j\right]+p_{j\rho}\mathbb{E}\left[\rho | s_j\right]\right) | s_i, s_j\right]\right] \\
&= \mathbb{E}\left[p_{i\rho}p_{j\rho}\mathbb{E}\left[\rho | s_i\right]\mathbb{E}\left[\rho | s_j\right]\right] + C_{i31} \\
&= \frac{\tau_{i,\rho}\tau_{j,\rho}p_{i\rho}p_{j\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho} + C_{i3}.
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[c_ip_j | s_i\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[c_ip_j | s_i, s_j\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\theta_i+\rho\right)\left(p_{j0}+p_{j\theta}\mathbb{E}\left[\theta_j | s_j\right]+p_{j\rho}\mathbb{E}\left[\rho | s_j\right]\right) | s_i, s_j\right]\right] \\
&= \mathbb{E}\left[p_{j\rho}\mathbb{E}\left[\rho | s_i\right]\mathbb{E}\left[\rho | s_j\right]\right] + C_{i41} \\
&= \frac{\tau_{i,\rho}\tau_{j,\rho}p_{j\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho} + C_{i4}.
\end{aligned}$$

Putting everything together, we are left with

$$
\begin{aligned}
\mathbb{E}\left[\pi_{i,1}\right] = & -\left(\frac{\tau_{i,\theta} p_{i\theta}^2}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta}+\frac{\tau_{i,\rho} p_{i\rho}^2}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho}\right) b+\left(\frac{\tau_{i,\theta} p_{i\theta}}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta}+\frac{\tau_{i,\rho} p_{i\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho}\right) b \\
& +\left(\frac{\tau_{i,\rho}\tau_{j,\rho} p_{i\rho} p_{j\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho}\right) e-\left(\frac{\tau_{i,\rho}\tau_{j,\rho} p_{j\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho}\right) e+C_i \\
= & \left(\frac{\left(1-p_{i\theta}\right) p_{i\theta}\tau_{i,\theta}}{\left(\tau_{i,\theta}+\tau_\theta\right)\tau_\theta}\right) b+\left(\frac{\left(1-p_{i\rho}\right) p_{i\rho}\tau_{i,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\tau_\rho}\right) b-\left(\frac{\left(1-p_{i\rho}\right) p_{j\rho}\tau_{i,\rho}\tau_{j,\rho}}{\left(\tau_{i,\rho}+\tau_\rho\right)\left(\tau_{j,\rho}+\tau_\rho\right)\tau_\rho}\right) e+C_i .
\end{aligned}
$$

Proof of Lemma 8 First period profits for given realization of signals $(s_\rho, s_{i,\theta})$ are

$$
\begin{aligned}
\pi_{i,1}(s_\rho, s_{i,\theta}) = & (a+e\mathbb{E}[p_{j,1}(s_\rho, s_{j,\theta})|s_\rho])(p_{i,1}^c(s_\rho, s_{i,\theta})-\mathbb{E}[c_i|s_\rho, s_{i,\theta}]) \\
& -b(p_{i,1}^c(s_\rho, s_{i,\theta}))^2+bp_{i,1}^c(s_\rho, s_{i,\theta})\mathbb{E}[c_i|s_\rho, s_{i,\theta}]
\end{aligned}
$$

Expected first period payoffs for given information precision levels[20] are

$$
\begin{aligned}
\mathbb{E}[\pi_{i,1}(s_p, s_{i,\theta})|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i] = & a\mathbb{E}[p_{i,1}^c(s_\rho, s_{i,\theta})-\mathbb{E}[c_i|s_\rho, s_{i,\theta}]|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i] \\
& +e\mathbb{E}[\mathbb{E}[p_{j,1}(s_\rho, s_{j,\theta})|s_\rho](p_{i,1}^c(s_\rho, s_{i,\theta})-\mathbb{E}[c_i|s_\rho, s_{i,\theta}])|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i] \\
& -b\mathbb{E}[(p_{i,1}^c(s_\rho, s_{i,\theta}))^2|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i]+b\mathbb{E}[p_{i,1}^c(s_\rho, s_{i,\theta})E[c_i|s_\rho, s_{i,\theta}]|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i]
\end{aligned}
$$

Grouping together the terms that do not depend on precision into C this ex-ante first period payoff becomes

$$
\begin{aligned}
\mathbb{E}[\pi_{i,1}(s_p, s_{i,\theta})|\tau_{\varepsilon,\rho}, \tau_{\varepsilon,\theta}^i] = & e(p_\rho^2-p_\rho)\mathbb{E}[\mathbb{E}[\rho|s_\rho]|\tau_{\varepsilon,\rho}]+b(p_\rho-p_\rho^2)\mathbb{E}[(\mathbb{E}[\rho|s_\rho])^2|\tau_{\varepsilon,\rho}] \\
& +b(p_\theta^2-p_\theta)\mathbb{E}[(\mathbb{E}[\theta_i|s_{i,\theta}])^2|\tau_{\varepsilon,\theta}^i]+C \\
= & b(p_\theta-p_\theta^2)\mathrm{Var}(\mathbb{E}[\theta_i|s_{i,\theta}])+(b-e)(p_\rho-p_\rho^2)\mathrm{Var}(\mathbb{E}[\rho|s_\rho])+C \\
= & b(p_\theta-p_\theta^2)\frac{\tau_{\varepsilon,\theta}^i}{(\tau_\theta+\tau_{\varepsilon,\theta}^i)\tau_\theta}+(b-e)(p_\rho-p_\rho^2)\frac{\tau_{\varepsilon,\rho}}{(\tau_\rho+\tau_{\varepsilon,\rho})\tau_\rho}+C
\end{aligned}
$$

B Conditional expectations

The following expectations are used throughout the paper.

[20] Firm i's payoffs are not affected by the precision of information firm j has about its private cost component

$$\mathbb{E}\left[p_{j,1}\middle|\, s_{i,\rho}\right] = p_{0,j} + \mu_\theta p_{\theta,j} + \bar{\tau}_{s,\rho,j}\mathbb{E}\left[\rho\middle|\, s_{i,\rho}\right] p_{\rho,j} + \left(1-\bar{\tau}_{s,\rho,j}\right)\mu_\rho p_{\rho,j} \tag{11}$$

B.1 Proofs of expectations

Proof of expectation (11).

$$\begin{aligned}
\mathbb{E}\left[p_{j,1}\middle|\, s_{i,\rho}\right] &= \mathbb{E}\left[p_{0,j} + \mathbb{E}\left[\theta_j\middle|\, s_{j,\theta}\right] p_{\theta,j} + \mathbb{E}\left[\rho\middle|\, s_{j,\rho}\right] p_{\rho,j}\middle|\, s_{i,\rho}\right] \\
&= p_{0,j} + \mathbb{E}\left[\bar{\tau}_{s,\theta,j}s_{j,\theta} + \left(1-\bar{\tau}_{s,\theta,j}\right)\mu_\theta\middle|\, s_{i,\rho}\right] p_{\theta,j} + \mathbb{E}\left[\bar{\tau}_{s,\rho,j}s_{j,\rho} + \left(1-\bar{\tau}_{s,\rho,j}\right)\mu_\rho\middle|\, s_{i,\rho}\right] \\
&= p_{0,j} + \mu_\theta p_{\theta,j} + \bar{\tau}_{s,\rho,j}\mathbb{E}\left[\rho\middle|\, s_{i,\rho}\right] p_{\rho,j} + \left(1-\bar{\tau}_{s,\rho,j}\right)\mu_\rho p_{\rho,j}
\end{aligned}$$

□

C Bounds on values

The following inequalities are used throughout the paper.

$$\beta \in \left[0, \frac{1}{3}\right] \tag{12}$$

$$\frac{b-e}{2b-e} \in \left[0, \frac{2}{3}\right] \tag{13}$$

$$\left(\frac{b-e}{2b-e}\right)\beta \in \left[0, \frac{2}{9}\right] \tag{14}$$

$$p_\theta \in \left[\frac{1}{3}, \frac{1}{2}\right] \tag{15}$$

$$\kappa \in [0,3] \tag{16}$$

$$\left|\frac{be}{4b^2-e^2}\right|\kappa \in [0,1] \tag{17}$$

$$\beta\kappa \in \left[0, \left|\frac{e}{b}\right|\right] \subseteq [0,1] \tag{18}$$

C.1 Proofs of bounds

Proof of inequality (12). Since $|e| \leq b$, $\beta = e^2/(4b^2-e^2) \geq 0$. To establish the upper bound, note that the numerator is increasing in e^2 and the denominator is decreasing in e^2, so the maximum value of β will be attained when e^2 is at its maximum. Since $e^2 \leq b^2$, it follows that $\beta \leq 3$. □

Proof of inequality (13). Since $|e| \leq b$, it is immediate that $(b-e)/(2b-e) \geq 0$. To establish the upper bound we examine the first derivative of the expression,[21]

$$\frac{-(2b-e)+(b-e)}{(2b-e)^2} = -\frac{b}{(2b-e)^2} < 0.$$

Then the derivative is negative everywhere, and the expression is maximized when e is at its minimum, $e = -b$. This gives

$$\frac{b-(-b)}{2b-(-b)} = \frac{2}{3}.$$

$\square$

Proof of inequality (14). This follows directly from inequalities (12) and (13). $\square$

Proof of inequalities (15) *and* (16). Since $\beta\kappa \geq 0$ and $p_\theta = 1/(2+\beta\kappa)$, it must be that $p_\theta \leq 1/2$. Further, p_θ will be minimized when $\beta\kappa$ is maximized. Looking at κ in isolation,

$$\kappa = \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\sigma_{s,\rho}^2 \bar{\tau}_{s,\rho}^2 p_\rho^2 + \left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right) \bar{\tau}_{s,\theta}^2 p_\theta^2}.$$

All involved terms are positive, so κ can be bounded above by assuming that $\bar{\tau}_{s,\rho} = 0$. This gives

$$\kappa \leq \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\left(\sigma_\theta^2 + \sigma_{s,\theta}^2\right) \bar{\tau}_{s,\theta}^2 p_\theta^2} = \frac{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta}{\sigma_\theta^2 \bar{\tau}_{s,\theta} p_\theta^2} = \frac{1}{p_\theta}.$$

Let $\underline{p}_\theta$ be the minimum feasible value of p_θ and $\overline{\beta} = 1/3$ be the maximum feasible value of β; then $\kappa \leq 1/\underline{p}_\theta$. It follows that

$$p_\theta \geq \frac{1}{2 + \frac{\beta}{\underline{p}_\theta}} \quad \Longrightarrow \quad \underline{p}_\theta \geq \frac{1}{2 + \frac{\overline{\beta}}{\underline{p}_\theta}}.$$

This gives

$$2\underline{p}_\theta + \overline{\beta} \geq 1 \quad \Longrightarrow \quad \underline{p}_\theta \geq \frac{1}{3}.$$

Then $p_\theta \geq 1/3$. It follows that $\kappa \leq 3$. Since $|e| \leq b$, $be/(4b^2 - e^2) \leq 1/3$, hence

$$\left(\frac{be}{4b^2 - e^2}\right)\kappa \leq \left(\frac{1}{3}\right) 3 = 1.$$

$\square$

[21] Basic intuition about fractions is sufficient for this maximization. We find that straightforward calculus is simpler to analyze.

Proof of inequality (17). ☐

Proof of inequality (18). Note that

$$\beta = \frac{e}{b}\left(\frac{be}{4b^2 - e^2}\right).$$

Then inequality (18) follows immediately from inequality (17). ☐