On the Merits of Optional Jury Service

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Remark: Please note that this is an on-going project. The pages that follow present the work we have done so far. It is not a complete paper. I will be happy to send you a copy of the paper as soon as it is completed. Meanwhile, I will appreciate your comments. Thank you.

Description: In the United States and in several other countries, jurors are chosen randomly from a list (for example voter list). The chosen jurors are required to serve on the jury. We propose and analyze an optional jury service (OJS) scheme and compare it with the mandatory jury service (MJS) scheme, where randomly chosen jurors are required to serve on the jury. Under OJS, a potential juror can opt out of jury duty with probability p by paying a fine. The randomly chosen jurors who decide to serve on the jury earn a payment for her service. The total amount of fine collected covers the total payments made to the jurors who serve, plus any administrative cost associated with the OJS.

- Each potential juror has a jury service cost c, distributed over an interval $[\underline{c}, \overline{c}]$. $g(c)$ is the density for individuals with jury service cost c, and $G(c)$ is the corresponding distribution function. We show that under an optimal OJS scheme:

- Individuals with extreme c values (that is, c values close to $\underline{c}$ or close to $\overline{c}$) prefer OJS to MJS. Individuals with moderate values of c prefer MJS to OJS.

- When $g(c)$ is uniformly distributed, a majority of individuals prefer OJS to MJS, if and only if OJS gives rise to a larger aggregate surplus (welfare) than MJS.

- When $g(c)$ is piecewise linear with an inverted V shape that is symmetric around the mean of $g(c)$, then a majority of individuals oppose OJS even if it gives rise to larger aggregate surplus (welfare) than MJS.

- When $g(c)$ is piecewise linear with a V shape that is symmetric around the mean of $g(c)$, then for $p > 0.75$, a majority of individuals oppose OJS even if it gives rise to larger aggregate surplus (welfare) than MJS. For $p < 0.75$, a majority of individuals prefer OJS even if it gives rise to smaller aggregate surplus (welfare) than MJS.

- When p is large and the mean of $g(c)$ is larger than the median of $g(c)$ then a majority of individuals oppose OJS even if it gives rise to larger aggregate surplus (welfare) than MJS.

- When p is large and the median of $g(c)$ is larger than the mean of $g(c)$ then a majority of individuals support OJS even if it gives rise to smaller aggregate surplus (welfare) than MJS.

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Notation

N = total mass of the continuum of individuals who are eligible for jury service.

$g(c)$ = density for individuals with jury service cost $c \in [\underline{c}, \overline{c}]$.

$G(c)$ = corresponding distribution function.

$\alpha(c)$ = social marginal value of the net financial benefit for an individual with cost c.

$v(c)$ = marginal utility of income for individual with cost c.

F = fee an individual can pay to request an exemption from jury service.

p = probability that requested jury service exemption is granted.

$\overline{p}$ = maximum feasible value of p (that satisfies the "trial by peers" requirement).[1]

w = wage paid to juror under an "opt-out" (OO) policy.

T = number of trials.

A = administrative cost of an OO policy (to collect F, disperse w, etc.). ($A > 0$.)

Assumptions

1. $\overline{p} > 0$.
2. $\alpha(c) \geq 0$ for all $c \in [\underline{c}, \overline{c}]$.
3. If an individual's request for jury service exemption is not honored, the fee paid (F) is returned to the individual.[2]
4. The utility of an individual with cost c who performs jury service under an OO policy is $v(c)\,[\,w - c\,]$.

[1] Some may object that an individual may be denied his right to a "trial by peers" if too many of his type opt out of jury service. To overcome this objection, we bound p below $\overline{p}$, where $\overline{p}$ reflects the maximum probability that a request to opt out of jury service is honored which ensures that all defendants receive a "trial by peers."

Substantial participation by all "types" may also be deemed necessary for successful operation of the judicial system. The $p \leq \overline{p}$ restriction also addresses this issue.

[2] We should be able to prove that this assumption is without essential loss of generality.

Opt-Out Considerations

Under an OO policy, a "type c" individual (i.e., one who incurs cost c if he performs jury service) will attempt to opt out if, when $\widetilde{N}$ individuals remain eligible for jury service:

$$p\,[\,-\,v(c)\,F\,] + [\,1-p\,]\,\frac{T}{\widetilde{N}}\,v(c)\,[\,w-c\,] \;\geq\; \frac{T}{\widetilde{N}}\,v(c)\,[\,w-c\,] \;\Leftrightarrow\; -\,p\,F \;\geq\; p\,\frac{T}{\widetilde{N}}\,[\,w-c\,]\,. \quad (1)$$

Under an OO policy, a type c individual will opt in if, when $\widetilde{N}$ individuals are eligible for jury service:

$$\frac{T}{\widetilde{N}}\,v(c)\,[\,w-c\,] \;\geq\; p\,[\,-\,v(c)\,F\,] + [\,1-p\,]\,\frac{T}{\widetilde{N}}\,v(c)\,[\,w-c\,] \;\Leftrightarrow\; p\,\frac{T}{\widetilde{N}}\,[\,w-c\,] \;\geq\; -\,p\,F\,. \quad (2)$$

Lemma 1. *Suppose $p > 0$, w, and F are such that some type $\widehat{c} \in [\,\underline{c}, \overline{c}\,]$ is indifferent between opting in and opting out. Then types $c \in [\,\underline{c},\, \widehat{c}\,)$ will opt in and types $c \in (\,\widehat{c}, \overline{c}\,]$ will opt out.*

Proof. (1) and (2) imply that if $p > 0$ and if $\widetilde{N} \in (0, N)$ individuals are eligible for jury service, then the type that is indifferent between opting in and opting out $(\widehat{c})$ is given by:

$$F \;=\; \frac{T}{\widetilde{N}}\,[\,\widehat{c} - w\,]\,. \quad (3)$$

Observe that when $p > 0$ and $\widetilde{N} \in (0, N)$, (1) will be satisfied for all $c \in [\,\widehat{c}, \overline{c}\,]$, whereas (2) will be satisfied for all $c \in [\,\underline{c},\, \widehat{c}\,]$. ■

Intuition. (Only) those for whom jury duty is particularly onerous will opt out.

Formulating the Social Problem

Lemma 1 implies that the (expected) number of individuals that are eligible for jury service is:

$$\widehat{N} \;\equiv\; N\,[\,G(\widehat{c}\,) + (\,1-p\,)\,(\,1 - G(\widehat{c}\,)\,)\,] \;=\; N\,[\,1 - p\,(\,1 - G(\widehat{c}\,)\,)\,]\,. \quad (4)$$

Therefore, from (3), the type $\widehat{c} \in [\,\underline{c}, \overline{c}\,]$ that is indifferent between opting in and opting out is given by:

$$T\,[\,\widehat{c} - w\,] \;=\; F\,\widehat{N}\,. \quad (5)$$

Observe from (4) that $\frac{d\widehat{N}}{d\widehat{c}} = N\,p\,g(\widehat{c}\,)$. Therefore, differentiating (5) provides:

$$[\,T - F\,N\,p\,g(\widehat{c}\,)\,]\,d\widehat{c} - T\,dw \;=\; 0 \;\Rightarrow\; \left.\frac{d\widehat{c}}{dw}\right|_{dF\,=\,dp\,=\,0} \;=\; \frac{T}{T - F\,N\,p\,g(\widehat{c}\,)}\,; \quad (6)$$

$$[\,T - F\,N\,p\,g(\widehat{c}\,)\,]\,d\widehat{c} - \widehat{N}\,dF \;=\; 0 \;\Rightarrow\; \left.\frac{d\widehat{c}}{dF}\right|_{dw\,=\,dp\,=\,0} \;=\; \frac{\widehat{N}}{T - F\,N\,p\,g(\widehat{c}\,)}\,; \quad (7)$$

$$[\,T - F\,N\,p\,g(\widehat{c}\,)\,]\,d\,\widehat{c} + F\,N\,[\,1 - G(\widehat{c}\,)\,]\;dp \;=\; 0$$

$$\Rightarrow \quad \left.\frac{d\,\widehat{c}}{dp}\right|_{dw\,=\,dF\,=\,0} \;=\; -\,\frac{F\,N\,[\,1 - G(\widehat{c}\,)\,]}{T - F\,N\,p\,g(\widehat{c}\,)}\,. \tag{8}$$

Expected social welfare (per capita) under an opt-out policy given $\widehat{c}$ is:

$$\begin{aligned}
W \;=\; & \int_{\underline{c}}^{\widehat{c}} \alpha(c)\,\frac{T}{\widehat{N}}\,v(c)\,[\,w - c\,]\,dG(c) \\
& \quad + \int_{\widehat{c}}^{\overline{c}} \alpha(c) \left\{ p\,[\,-v(c)F\,] + [\,1 - p\,]\,\frac{T}{\widehat{N}}\,v(c)\,[\,w - c\,] \right\} dG(c) - \frac{A}{N} \\
=\; & \frac{T}{\widehat{N}} \int_{\underline{c}}^{\overline{c}} \alpha(c)\;v(c)\,[\,w - c\,]\,dG(c) - \frac{T}{\widehat{N}} \int_{\widehat{c}}^{\overline{c}} \alpha(c)\;v(c)\,[\,w - c\,]\,dG(c) \\
& \quad - p\,F \int_{\widehat{c}}^{\overline{c}} \alpha(c)\;v(c)\,dG(c) \;+ [\,1 - p\,]\,\frac{T}{\widehat{N}} \int_{\widehat{c}}^{\overline{c}} \alpha(c)\;v(c)\,[\,w - c\,]\,dG(c) - \frac{A}{N} \\
=\; & \frac{T}{\widehat{N}} \int_{\underline{c}}^{\overline{c}} \alpha(c)\;v(c)\,[\,w - c\,]\,dG(c) - p \int_{\widehat{c}}^{\overline{c}} \alpha(c)\;v(c) \left\{ F + \frac{T}{\widehat{N}}\,[\,w - c\,] \right\} dG(c) - \frac{A}{N}\,.
\end{aligned} \tag{9}$$

An opt-out policy in which only types $c \in [\,\widehat{c}, \overline{c}\,]$ attempt to opt out will be self-financing in the sense that the expected payments to jurors and the administrative cost (A) do not exceed the expected revenue from opt-out fees if:

$$p\,[\,1 - G(\widehat{c}\,)\,]\,N\,F \;\geq\; T\,w + A\,. \tag{10}$$

Observe that the expression in (9) can be written as:

$$\frac{T}{\widehat{N}}\,Z - p\,F \int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,dG(c) - \frac{A}{N} \tag{11}$$

where:

$$Z \;\equiv\; \int_{\underline{c}}^{\widehat{c}} \alpha(c)\,v(c)\,[\,w - c\,]\,dG(c) + [\,1 - p\,] \int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,[\,w - c\,]\,dG(c)\,. \tag{12}$$

(10) and (12) imply that [P], the social problem under an opt-out policy, is:

$$\underset{w,\,F,\,p\,\in\,[\,0,\,\overline{p}\,]}{Maximize} \quad \frac{T}{\widehat{N}}\,Z \;-\; p\,F \int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,dG(c) - \frac{A}{N}$$

subject to:

$$p\,[\,1 - G(\widehat{c}\,)\,]\,N\,F \;\geq\; T\,w + A\,, \quad \text{and} \tag{13}$$

$$\widehat{N} \;=\; N\,[\,1 - p\,(\,1 - G(\widehat{c}\,)\,)\,] \;\geq\; T\,, \tag{14}$$

where $\widehat{c}$ is defined by (5).

Solving the Social Problem

Let λ_f denote the Lagrange multiplier associated with the "self-financing" constraint (13), and let λ_t denote the Lagrange multiplier associated with the "adequate jury pool" constraint, (14). Then the necessary conditions for a solution to [P] include:

$$\begin{aligned}
F: \quad & -\,p \int_{\widehat{c}}^{\bar{c}} \alpha(c)\,v(c)\,dG(c) - \frac{T\,Z}{\left(\widehat{N}\right)^2}\,N\,p\,g(\widehat{c}\,)\,\frac{d\,\widehat{c}}{dF} \\
& \quad + \frac{T}{\widehat{N}}\,p\,\alpha(\widehat{c}\,)\,v(\widehat{c}\,)\,[\,w - \widehat{c}\,]\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dF} + p\,F\,\alpha(\widehat{c}\,)\,v(\widehat{c}\,)\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dF} \\
& \quad + \lambda_f\,p\,[\,1 - G(\widehat{c}\,)\,]\,N - \lambda_f\,p\,N\,F\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dF} + \lambda_t\,p\,N\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dF} \;=\; 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & -\,p \int_{\widehat{c}}^{\bar{c}} \alpha(c)\,v(c)\,dG(c) + \lambda_f\,p\,[\,1 - G(\widehat{c}\,)\,]\,N - \;p\,N\,g(\widehat{c}\,)\,\frac{d\,\widehat{c}}{dF}\left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\,F - \lambda_t\right] \\
& \quad + \;\alpha(\widehat{c}\,)\,v(\widehat{c}\,)\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dF}\,p\left[\frac{T}{\widehat{N}}\,(\,w - \widehat{c}\,) + F\right] \;=\; 0\,;
\end{aligned} \tag{15}$$

$$\begin{aligned}
w: \quad & \frac{T}{\widehat{N}}\left[\int_{\underline{c}}^{\bar{c}} \alpha(c)\,v(c)\,dG(c) - p \int_{\widehat{c}}^{\bar{c}} \alpha(c)\,v(c)\,dG(c)\right] - \frac{T\,Z}{\left(\widehat{N}\right)^2}\,N\,p\,g(\widehat{c}\,)\,\frac{d\,\widehat{c}}{dw} \\
& \quad + \frac{T}{\widehat{N}}\,p\,\alpha(\widehat{c}\,)\,v(\widehat{c}\,)\,[\,w - \widehat{c}\,]\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dw} + p\,F\,\alpha(\widehat{c}\,)\,v(\widehat{c}\,)\,g(\widehat{c}\,)\,\frac{d\widehat{c}}{dw} \\
& \quad - \lambda_f\,T - \lambda_f\,p\,N\,F\,g(\widehat{c})\,\frac{d\widehat{c}}{dw} + \lambda_t\,p\,N\,g(\widehat{c})\,\frac{d\widehat{c}}{dw} \;=\; 0
\end{aligned}$$

$$\Rightarrow \quad \int_{\underline{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) - p \int_{\widehat{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) - \lambda_f\, \widehat{N}$$

$$- \;\; p\, N\, g(\widehat{c})\, \frac{d\widehat{c}}{dw}\, \frac{\widehat{N}}{T} \left[\frac{T\, Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \right]$$

$$+ \;\; \alpha(\widehat{c})\, v(\widehat{c})\, g(\widehat{c})\, \frac{d\widehat{c}}{dw}\, \frac{\widehat{N}}{T}\, p \left[\frac{T}{\widehat{N}} (\, w - \widehat{c}\,) + F \right] \;=\; 0\,. \tag{16}$$

Using (6) and (7), (16) can be written as:

$$\int_{\underline{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) - p \int_{\widehat{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) - \lambda_f\, \widehat{N} - p\, N\, g(\widehat{c})\, \frac{d\widehat{c}}{dF} \left[\frac{T\, Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \right]$$

$$+ \;\; \alpha(\widehat{c})\, v(\widehat{c})\, g(\widehat{c})\, \frac{d\widehat{c}}{dF}\, p \left[\frac{T}{\widehat{N}} (\, w - \widehat{c}\,) + F \right] \;=\; 0\,. \tag{17}$$

Subtracting (15) from (17) and using (5) provides:

$$\int_{\underline{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) - \lambda_f \left[\widehat{N} + p\, (\, 1 - G(\widehat{c})\,)\, N \right] \;=\; 0$$

$$\Rightarrow \quad \lambda_f \;=\; \frac{1}{N} \int_{\underline{c}}^{\bar{c}} \alpha(c)\, v(c)\, dG(c) \;>\; 0\,. \tag{18}$$

Because the self-financing constraint binds ($\lambda_f > 0$), (13) implies:

$$p\, [\, 1 - G(\widehat{c})\,]\, N\, F \;=\; T\, w + A \quad \Rightarrow \quad w \;=\; p\, [\, 1 - G(\widehat{c})\,]\, \frac{N}{T}\, F - \frac{A}{T}\,. \tag{19}$$

Also, from (5):

$$F \;=\; \frac{T}{\widehat{N}}\, [\, \widehat{c} - w\,]\,. \tag{20}$$

Combining (19) and (20) and using (4) provides:

$$w \;=\; p\, [\, 1 - G(\widehat{c})\,]\, \frac{N}{\widehat{N}}\, [\, \widehat{c} - w\,] - \frac{A}{T}$$

$$\Rightarrow \quad w \left[1 + p\, (\, 1 - G(\widehat{c})\,)\, \frac{N}{\widehat{N}} \right] \;=\; p\, [\, 1 - G(\widehat{c})\,]\, \frac{N}{\widehat{N}}\, \widehat{c} - \frac{A}{T}$$

$$\Rightarrow \quad w \left[\widehat{N} + p\, (\, 1 - G(\widehat{c})\,)\, N \right] \;=\; p\, [\, 1 - G(\widehat{c})\,]\, N\, \widehat{c} - \frac{A}{T}\, \widehat{N}$$

$$\Rightarrow \quad w \;=\; p\, [\, 1 - G(\widehat{c})\,]\; \widehat{c} - \frac{A}{N}\, \frac{\widehat{N}}{T}\,. \tag{21}$$

(5) and (21) imply:

$$\widehat{c} - w \;=\; [\,1 - p\,(\,1 - G(\widehat{c}\,)\,)\,]\,\widehat{c} + \frac{A}{N}\,\frac{\widehat{N}}{T} \;=\; \frac{\widehat{N}}{N}\left[\,\widehat{c} + \frac{A}{T}\,\right]. \tag{22}$$

(20) and (22) provide:

$$F \;=\; \frac{T}{N}\left[\,\widehat{c} + \frac{A}{T}\,\right]. \tag{23}$$

Letting λ_p denote the Lagrange multiplier associated with the constraint $p \leq \overline{p}$, the necessary condition for an optimum with respect to p is:

$$\begin{aligned} p: \quad & -\frac{T}{\widehat{N}}\int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,[\,w - c\,]\,dG(c) - F\int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,dG(c) - \frac{T\,Z}{\left(\widehat{N}\right)^2}\left[\frac{\partial \widehat{N}}{\partial p} + \frac{\partial \widehat{N}}{\partial \widehat{c}}\,\frac{d\widehat{c}}{dp}\right] \\ & + \frac{T}{\widehat{N}}\,p\,\alpha(\,\widehat{c}\,)\,v(\,\widehat{c}\,)\,[\,w - \widehat{c}\,]\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp} + p\,F\,\alpha(\,\widehat{c}\,)\,v(\,\widehat{c}\,)\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp} \\ & + \lambda_f\,N\,F\left[\,1 - G(\,\widehat{c}\,) - p\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp}\,\right] + \lambda_t\left[\frac{\partial \widehat{N}}{\partial p} + \frac{\partial \widehat{N}}{\partial \widehat{c}}\,\frac{d\widehat{c}}{dp}\right] - \lambda_p \;=\; 0\,. \end{aligned} \tag{24}$$

Using (20), (24) can be written as:

$$\begin{aligned} p: \quad & -\frac{T}{\widehat{N}}\int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,[\,w - c + \widehat{c} - w\,]\,dG(c) - \frac{T\,Z}{\left(\widehat{N}\right)^2}\left[-N\,[\,1 - G(\,\widehat{c}\,)\,] + p\,N\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp}\right] \\ & + \alpha(\,\widehat{c}\,)\,v(\,\widehat{c}\,)\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp}\,p\left[-\frac{T}{\widehat{N}}\,(\,\widehat{c} - w\,) + F\right] \\ & + \lambda_f\,N\,F\,[\,1 - G(\,\widehat{c}\,)\,] - \lambda_t\,N\,[\,1 - G(\,\widehat{c}\,)\,] - \lambda_f\,N\,F\,p\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp} \\ & + \lambda_t\,p\,N\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp} - \lambda_p \;=\; 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & \frac{T}{\widehat{N}}\int_{\widehat{c}}^{\overline{c}} \alpha(c)\,v(c)\,[\,c - \widehat{c}\,]\,dG(c) + N\,[\,1 - G(\,\widehat{c}\,)\,]\left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\,F - \lambda_t\right] \\ & \qquad - p\,N\,g(\,\widehat{c}\,)\,\frac{d\widehat{c}}{dp}\left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\,F - \lambda_t\right] - \lambda_p \;\leq\; 0 \end{aligned}$$

$$\Rightarrow \quad \frac{T}{\widehat{N}} \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c)\, [\, c \,-\, \widehat{c}\,]\, dG(c) - \frac{d\widehat{N}}{dp} \left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \right] - \lambda_p \;=\; 0 \tag{25}$$

where

$$\frac{d\widehat{N}}{dp} \;=\; -\, N\, [\, 1 - G(\,\widehat{c}\,)\,] + p\, N\, g(\,\widehat{c}\,)\, \frac{d\widehat{c}}{dp}\,.$$

From (15), (18), and (20):

$$-\, p \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) + \lambda_f\, p\, [\, 1 - G(\,\widehat{c}\,)\,]\, N \;=\; p\, N\, g(\,\widehat{c}\,)\, \frac{d\widehat{c}}{dF} \left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \right]$$

$$\begin{aligned} \Rightarrow \quad & p\, N\, g(\,\widehat{c}\,)\, \frac{d\widehat{c}}{dF} \left[\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \right] \\ & \quad = \; p\, [\, 1 - G(\,\widehat{c}\,)\,] \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) - p \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) \\ & \quad = \; p \left[\int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c)\, dG(c) - G(\,\widehat{c}\,) \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) \right]. \end{aligned} \tag{26}$$

(26) implies that if $p > 0$ and $\frac{d\widehat{c}}{dF}$ is well-defined, then:

$$\frac{T\,Z}{\left(\widehat{N}\right)^2} + \lambda_f\, F - \lambda_t \;=\; \frac{\int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c)\, dG(c) - G(\,\widehat{c}\,) \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c)}{N\, g(\,\widehat{c}\,)\, \frac{d\widehat{c}}{dF}}\,. \tag{27}$$

Lemma 2. *Suppose* $\alpha(c) = v(c) = 1$ *for all* $c \in [\,\underline{c},\, \overline{c}\,]$*. Also suppose* $p > 0$ *and* $\frac{d\widehat{c}}{dF}$ *is well-defined. Then* $\frac{T\,Z}{(\widehat{N})^2} + \lambda_f\, F - \lambda_t \,=\, 0$ *at the solution to [P].*

<u>Proof</u>. When $\alpha(c) = v(c) = 1$ for all $c \in [\,\underline{c},\, \overline{c}\,]$:

$$\int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c)\, dG(c) \;=\; G(\,\widehat{c}\,) \;=\; G(\,\widehat{c}\,) \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c)\,.$$

Therefore, the conclusion follows immediately from (27). ■

Conclusion 1. *Suppose the conditions of Lemma 2 hold and* $G(\widehat{c}\,) < 1$. *Then* $p = \overline{p}$ *at the solution to [P].*

Proof. (25) and Lemma 2 imply that under the specified conditions, $\lambda_p > 0$. Therefore, $p = \overline{p}$, by complementary slackness. ■

Interpretation. When the social objective is to maximize aggregate welfare, it is optimal to allow anyone who would like to opt out of jury duty do so. Limiting an individual's ability to opt out eliminates an opportunity to replace a high-cost juror with a low-cost juror

Calculating Expected Net Payoffs

From (21) and (23), the expected net payoff of a type $c \in (\widehat{c}\,, \overline{c}\,]$ individual is:

$$
\begin{aligned}
u(c) \;&=\; p\,[\,-\,v(c)\,F\,] + [\,1-p\,]\,\frac{T}{\widehat{N}}\,v(c)\,[\,w-c\,] \\
&=\; -\,p\,v(c)\,\frac{T}{N}\left[\widehat{c} + \frac{A}{T}\right] + [\,1-p\,]\,\frac{T}{\widehat{N}}\,v(c)\left[p\,[\,1-G(\widehat{c}\,)\,]\,\widehat{c} - \frac{A}{N}\frac{\widehat{N}}{T} - c\right] \\
&=\; \widehat{c}\,p\,v(c)\,T\left[\left(\frac{1-p}{\widehat{N}}\right)[\,1-G(\widehat{c}\,)\,] - \frac{1}{N}\right] - [\,1-p\,]\,\frac{T}{\widehat{N}}\,v(c)\,c \\
&\qquad\qquad -\,p\,v(c)\,\frac{A}{N} - [\,1-p\,]\,v(c)\,\frac{A}{N} \\
&=\; p\,v(c)\,\frac{T}{N\,\widehat{N}}\left[\,(\,1-p\,)\,[\,1-G(\widehat{c}\,)\,]\,N - \widehat{N}\,\right]\widehat{c} - [\,1-p\,]\,\frac{T}{\widehat{N}}\,v(c)\,c - v(c)\,\frac{A}{N} \\
&=\; p\,v(c)\,\frac{T}{N\,\widehat{N}}\,[\,-G(\widehat{c}\,)\,N\,]\,\widehat{c} - [\,1-p\,]\,\frac{T}{\widehat{N}}\,v(c)\,c - v(c)\,\frac{A}{N} \\
&=\; -\,v(c)\left[\frac{T}{\widehat{N}}\,[\,p\,G(\widehat{c}\,)\,\widehat{c} + (\,1-p\,)\,c\,] + \frac{A}{N}\right].
\end{aligned} \tag{28}
$$

The fifth equality in (28) holds because, from (4):

$$
\begin{aligned}
&[\,1-p\,]\,[\,1-G(\widehat{c}\,)\,]\,N - \widehat{N} \;=\; [\,1-p\,]\,[\,1-G(\widehat{c}\,)\,]\,N - N\,[\,1-p\,(\,1-G(\widehat{c}\,)\,)\,] \\
&=\; N\,\{\,[\,1-p\,]\,[\,1-G(\widehat{c}\,)\,] - 1 + p\,[\,1-G(\widehat{c}\,)\,]\,\} \;=\; N\,[\,1-G(\widehat{c}\,)-1\,] \;=\; -\,N\,G(\widehat{c}\,)\,.
\end{aligned}
$$

From (21) and using (4), the expected net payoff of a type $c \in [\,\underline{c}\,, \widehat{c}\,]$ individual is:

$$
u(c) \;=\; \frac{T}{\widehat{N}}\,v(c)\left[p\,[\,1-G(\widehat{c}\,)\,]\,\widehat{c} - \frac{A}{N}\frac{\widehat{N}}{T} - c\right]
$$

$$= \; v(c) \left[\frac{T}{\widehat{N}} \left[p \, (1 - G(\widehat{c})) \, \widehat{c} \; - \; c \right] - \frac{A}{N} \right] . \tag{29}$$

Conclusion 2. *The individuals whose expected net payoff increases when an optimal opt-out policy is implemented are those for whom* $c > \widehat{c} + \frac{A}{T} \frac{\widehat{N}}{\overline{p} G(\widehat{c}) N}$ *or* $c < \widehat{c} - \frac{A}{T} \frac{\widehat{N}}{\overline{p} [1 - G(\widehat{c})] N}$.

Proof. From (28) and using (4), the expected net payoff of a type $c \in (\widehat{c}, \overline{c}]$ individual increases when an optimal opt-out policy is implemented if:

$$- v(c) \left[\frac{T}{\widehat{N}} \left[\overline{p} \, G(\widehat{c}) \, \widehat{c} + (1 - \overline{p}) \; c \right] + \frac{A}{N} \right] \;\; > \;\; - \, v(c) \, \frac{T}{N} \, c$$

$$\Leftrightarrow \;\; \frac{T}{N} \, c \; > \; \frac{T}{\widehat{N}} \left[\overline{p} \, G(\widehat{c}) \, \widehat{c} + (1 - \overline{p}) \; c \right] + \frac{A}{N}$$

$$\Leftrightarrow \;\; \frac{T}{\widehat{N} \, N} \, c \left[\, \widehat{N} - N \, (1 - \overline{p}) \right] \;\; > \;\; \frac{T}{\widehat{N} \, N} \, \overline{p} \, N \, G(\widehat{c}) \, \widehat{c} + \frac{A \, \widehat{N}}{\widehat{N} \, N}$$

$$\Leftrightarrow \;\; c \left[\, N \, G(\widehat{c}) + N \, (1 - \overline{p}) \, (1 - G(\widehat{c}) \,) - N \, (1 - \overline{p}) \right] \;\; > \;\; \overline{p} \, N \, G(\widehat{c}) \, \widehat{c} + \frac{A \, \widehat{N}}{T}$$

$$\Leftrightarrow \;\; \overline{p} \, N \, G(\widehat{c}) \, c \;\; > \;\; \overline{p} \, N \, G(\widehat{c}) \, \widehat{c} + \frac{A \, \widehat{N}}{T} \quad \Leftrightarrow \quad c \; > \; \widehat{c} + \frac{A}{T} \, \frac{\widehat{N}}{\overline{p} \; G(\widehat{c}) \, N} \, .$$

From (29) and using (4), the expected net payoff of a type $c \in [\, \underline{c}, \widehat{c} \,]$ individual increases when an optimal opt-out policy is implemented if:

$$v(c) \left[\frac{T}{\widehat{N}} \left[\overline{p} \, (1 - G(\widehat{c}) \,) \, \widehat{c} \; - \; c \right] - \frac{A}{N} \right] \;\; > \;\; - \, v(c) \, \frac{T}{N} \, c$$

$$\Leftrightarrow \;\; \frac{T}{N} \, c \; > \; \frac{T}{\widehat{N}} \left[\, c - \overline{p} \, (1 - G(\widehat{c}) \,) \, \widehat{c} \, \right] + \frac{A}{N}$$

$$\Leftrightarrow \;\; \frac{T}{\widehat{N} \, N} \left[\, \widehat{N} \, - N \right] c \;\; > \;\; - \, \frac{T \, N}{\widehat{N} \, N} \, \overline{p} \left[1 - G(\widehat{c}) \right] \widehat{c} \; + \frac{A \, \widehat{N}}{\widehat{N} \, N}$$

$$\Leftrightarrow \;\; \left[\, N - G(\widehat{c}) \, N - (1 - \overline{p}) \, (1 - G(\widehat{c}) \,) \, N \, \right] c \;\; < \;\; \overline{p} \, N \left[1 - G(\widehat{c}) \right] \widehat{c} \; - \; \frac{A \, \widehat{N}}{T}$$

$$\Leftrightarrow \;\; \overline{p} \, N \left[1 - G(\widehat{c}) \right] c \;\; < \;\; \overline{p} \, N \left[1 - G(\widehat{c}) \right] \widehat{c} \; - \frac{A \, \widehat{N}}{T}$$

$$\Leftrightarrow \;\; c \;\; < \;\; \widehat{c} - \frac{A}{T} \, \frac{\widehat{N}}{\overline{p} \left[1 - G(\widehat{c}) \, \right] N} \, . \qquad \blacksquare$$

Conclusion 2 implies that in the absence of administrative costs, all individuals gain when an optimal opt-out policy is implemented. The gains arise because jury duty is performed by low-cost individuals in place of high-cost individuals, thereby reducing aggregate costs.

When $A > 0$, those who gain from the OO policy are those with the highest and the lowest costs of performing jury service.

Those with the highest c's gain because everyone who opts out is charged F and those with the highest c's avoid the highest costs.

Those with the lowest c's gain because everyone who performs jury service is paid the same amount (w), so the net gain ($w - c$) is largest for those with the lowest costs.

Expected Social Welfare

(28) and (29) imply that expected social welfare per capita, given p, is:

$$
\begin{aligned}
W \;=\; \int_{\underline{c}}^{\overline{c}} \alpha(c)\, u(c)\, dG(c) \;=\; & \int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c) \left[\frac{T}{\widehat{N}} \left[p\,(1 - G(\widehat{c})\,)\; \widehat{c} \,-\, c \right] - \frac{A}{N} \right] dG(c) \\
& - \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c) \left[\frac{T}{\widehat{N}} \left[p\, G(\widehat{c})\; \widehat{c} + (\,1 - p\,)\; c \right] + \frac{A}{N} \right] dG(c)
\end{aligned}
$$

$$
\begin{aligned}
=\; & -\, p\, \widehat{c}\, G(\widehat{c})\, \frac{T}{\widehat{N}} \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) \;+ p\, \widehat{c}\, \frac{T}{\widehat{N}} \int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c)\, dG(c) \\
& -\; \frac{T}{\widehat{N}} \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, c\; dG(c) + p\, \frac{T}{\widehat{N}} \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c)\, c\; dG(c) - \frac{A}{N} \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) \\
=\; & p\, \widehat{c}\, \frac{T}{\widehat{N}} \left[\int_{\underline{c}}^{\widehat{c}} \alpha(c)\, v(c)\, dG(c) - G(\widehat{c}) \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c) \right] \\
& +\; p\, \frac{T}{\widehat{N}} \int_{\widehat{c}}^{\overline{c}} \alpha(c)\, v(c)\, c\; dG(c) - \frac{T}{\widehat{N}} \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, c\; dG(c) - \frac{A}{N} \int_{\underline{c}}^{\overline{c}} \alpha(c)\, v(c)\, dG(c)\,. \qquad (30)
\end{aligned}
$$

A special case of interest is the case in which all individuals have the same, constant marginal utility of income (so $v(c) = 1$ for all $c \in [\,\underline{c},\, \overline{c}\,]$) and society values identically the net benefits that accrue to all individuals (so $\alpha(c) = 1$ for all $c \in [\,\underline{c},\, \overline{c}\,]$).

Assumption 1. $\alpha(c) \,=\, v(c) \,=\, 1$ for all $c \in [\,\underline{c},\, \overline{c}\,]$.

Maximizing Aggregate Surplus

(30) implies that if Assumption 1 holds, then:

$$\begin{aligned} W(p) &= p\,\widehat{c}\,\frac{T}{\widehat{N}}\left[\int_{\underline{c}}^{\widehat{c}} dG(c) - G(\widehat{c})\int_{\underline{c}}^{\overline{c}} dG(c)\right] \\ &\qquad + \frac{T}{\widehat{N}}\left[p\int_{\widehat{c}}^{\overline{c}} c\, dG(c) - \int_{\underline{c}}^{\overline{c}} c\, dG(c)\right] - \frac{A}{N}\int_{\underline{c}}^{\overline{c}} dG(c) \\ &= \frac{T}{\widehat{N}}\left[p\int_{\widehat{c}}^{\overline{c}} c\, dG(c) - \int_{\underline{c}}^{\overline{c}} c\, dG(c)\right] - \frac{A}{N} \\ &= -\frac{T}{\widehat{N}}\left[\int_{\underline{c}}^{\widehat{c}} c\, dG(c) + [\,1-p\,]\int_{\widehat{c}}^{\overline{c}} c\, dG(c)\right] - \frac{A}{N}\,. \end{aligned} \tag{31}$$

(31) indicates that aggregate welfare per person is simply the negative of the sum of: (i) per-person administrative costs ($\frac{A}{N}$); and (ii) expected jury service cost. Expected jury service cost is the product of the probability of being called for jury service ($\frac{T}{\widehat{N}}$) and $E\{\,c\,|\,I\,;\,\widehat{c}\}$, the expected personal cost of an individual who is in the jury pool, i.e.,

$$E\{\,c\,|\,I\,;\,\widehat{c}\} \;\equiv\; \int_{\underline{c}}^{\widehat{c}} c\, dG(c) + [\,1-\overline{p}\,]\int_{\widehat{c}}^{\overline{c}} c\, dG(c)\,. \tag{32}$$

(31) implies that maximizing average aggregate surplus is equivalent to minimizing

$$-W \;=\; \frac{T}{\widehat{N}}\left[\int_{\underline{c}}^{\widehat{c}} c\, dG(c) + [\,1-\overline{p}\,]\int_{\widehat{c}}^{\overline{c}} c\, dG(c)\right] + \frac{A}{N}\,. \tag{33}$$

We will focus on the setting where $\overline{p} < 1$ and $N > \frac{T}{1-\overline{p}}$. In this case, N is sufficiently large to ensure that the adequate jury pool constraint (14) does not bind.

Assumption 2. $\overline{p} < 1$, and $N > \frac{T}{1-\overline{p}}$.

Lemma 3. *Suppose Assumptions 1 and 2 hold. Then average surplus is maximized when* $-W$ *is minimized with respect to* $\widehat{c}$.

Proof. If $\overline{p} < 1$, then $1 - p[1 - G(\widehat{c})] > 0$ for all $p \leq \overline{p}$ and $\widehat{c}$. Therefore, $\frac{T}{1-p[1-G(\widehat{c})]}$ is a finite number. Hence, if N exceeds $\frac{T}{1-\overline{p}}$ (which weakly exceeds $\frac{T}{1-p[1-G(\widehat{c})]}$ for all $p \leq \overline{p}$), then (14) holds as a strict inequality. When (14) does not bind, average surplus is maximized when $\widehat{c}$ is chosen to minimize $-W$. If the optimal $\widehat{c}$ lies in $(\underline{c}, \overline{c})$, then the corresponding F and w are uniquely determined by (3), (4), and $p[1 - G(\widehat{c})] N F = T w + A$ (which is (13) with equality), with $p = \overline{p}$. (Note that Conclusion 1 holds for large N.) ■

Conclusion 3. *If Assumptions 1 and 2 hold, then* $-\widetilde{W}$ *is minimized at* $\widehat{c}^*$, *where*

$$\widehat{c}^* = \frac{\delta(\widehat{c}^*)}{\beta(\widehat{c}^*)} = \frac{\int_{\underline{c}}^{\widehat{c}^*} c\, dG(c) + [1-\overline{p}] \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)}{G(\widehat{c}^*) + [1-\overline{p}][1 - G(\widehat{c}^*)]} . \tag{34}$$

Proof. To minimize $-W$ with respect to $\widehat{c}$, observe from (14) and (33) that:

$$-W = \frac{T}{N} \left[\frac{\int_{\underline{c}}^{\widehat{c}} c\, dG(c) + [1-\overline{p}] \int_{\widehat{c}}^{\overline{c}} c\, dG(c)}{G(\widehat{c}) + (1-\overline{p})(1 - G(\widehat{c}))} \right] + \frac{A}{N} . \tag{35}$$

(35) and Claim 3 imply that to maximize average surplus, it suffices to minimize:

$$-\widetilde{W} = \frac{\delta(\widehat{c})}{\beta(\widehat{c})} \tag{36}$$

where:

$$\delta(\widehat{c}) \equiv \int_{\underline{c}}^{\widehat{c}} c\, dG(c) + [1-\overline{p}] \int_{\widehat{c}}^{\overline{c}} c\, dG(c) \quad \text{and} \quad \beta(\widehat{c}) \equiv G(\widehat{c}) + [1-\overline{p}][1 - G(\widehat{c})] . \tag{37}$$

From (36):

$$\log\left(-\widetilde{W}\right) = \log(\delta(\widehat{c})) - \log(\beta(\widehat{c}))$$

$$\Rightarrow \frac{\partial}{\partial \widehat{c}} \left\{ \log\left(-\widetilde{W}\right) \right\} = \frac{\delta'(\widehat{c})}{\delta(\widehat{c})} - \frac{\beta'(\widehat{c})}{\beta(\widehat{c})} . \tag{38}$$

From (37):

$$\delta'(\widehat{c}) = \widehat{c}\, g(\widehat{c}) - [1-\overline{p}]\widehat{c}\, g(\widehat{c}) = \overline{p}\, \widehat{c}\, g(\widehat{c}) , \text{ and}$$

$$\beta'(\widehat{c}) = g(\widehat{c}) - [1-\overline{p}]\, g(\widehat{c}) = \overline{p}\, g(\widehat{c}) . \tag{39}$$

(38) and (39) imply:

$$\frac{\partial}{\partial \widehat{c}} \left\{ \log \left(- \widetilde{W} \right) \right\} = \overline{p}\, g(\widehat{c}) \left[\frac{\widehat{c}}{\delta(\widehat{c})} - \frac{1}{\beta(\widehat{c})} \right] = \frac{\overline{p}\, g(\widehat{c}) \left[\widehat{c}\, \beta(\widehat{c}) - \delta(\widehat{c}) \right]}{\delta(\widehat{c})\, \beta(\widehat{c})}. \tag{40}$$

Define $\gamma(\widehat{c}) \equiv \widehat{c}\, \beta(\widehat{c}) - \delta(\widehat{c})$. Differentiating $\gamma(\widehat{c})$, using (39), provides:

$$\frac{\partial \gamma(\widehat{c})}{\partial \widehat{c}} = \beta(\widehat{c}) + \widehat{c}\, \overline{p}\, g(\widehat{c}) - \overline{p}\, \widehat{c}\, g(\widehat{c}) = \beta(\widehat{c}) > 0. \tag{41}$$

Also, from (37):

$$\gamma(\underline{c}) = \underline{c}\, \beta(\underline{c}) - \delta(\underline{c}) = \underline{c}\, [1 - \overline{p}] - E\{c\} [1 - \overline{p}] = [1 - \overline{p}] [\underline{c} - E\{c\}] < 0;$$

$$\gamma(\overline{c}) = \overline{c}\, \beta(\overline{c}) - \delta(\overline{c}) = \overline{c} - E\{c\} > 0. \tag{42}$$

(41) and (42) imply that there exists a unique $\widehat{c}^* \in (\underline{c}, \overline{c})$ such that: (i) $\gamma(\widehat{c}) < 0$ for $\widehat{c} < \widehat{c}^*$; (ii) $\gamma(\widehat{c}^*) = 0$; and (iii) $\gamma(\widehat{c}) > 0$ for $\widehat{c} > \widehat{c}^*$. Therefore, (40) implies that $-\widetilde{W}$ is minimized at $\widehat{c}^*$. ■

Explanation. Reducing $\widehat{c}$ below $\overline{c}$ has two countervailing effects on expected welfare. First, reducing $\widehat{c}$ increases expected welfare by reducing the expected personal cost of those called for jury service, i.e., from (32):

$$\frac{\partial E\{c \,|\, I; \widehat{c}\}}{\partial \widehat{c}} = \widehat{c}\, [1 - (1 - \overline{p})]\, g(\widehat{c}) = \widehat{c}\, \overline{p}\, g(\widehat{c}) > 0. \tag{43}$$

Second, reducing $\widehat{c}$ reduces expected welfare by increasing the probability of being called for jury service ($\frac{T}{\widehat{N}}$) by reducing the size of the potential jury pool, i.e., from (4):

$$\frac{\partial \widehat{N}}{\partial \widehat{c}} = N\, \overline{p}\, g(\widehat{c}) > 0. \tag{44}$$

$\widehat{c}^*$ is the value of $\widehat{c}$ that balances these two countervailing effects by minimizing $\frac{T}{\widehat{N}}\, E\{c \,|\, I; \widehat{c}\}$. Specifically, from (43) and (44):

$$\frac{\partial}{\partial \widehat{c}} \frac{E\{c \,|\, I; \widehat{c}\}}{\widehat{N}} \stackrel{s}{=} \widehat{N}\, \frac{\partial E\{c \,|\, I; \widehat{c}\}}{\partial \widehat{c}} - \frac{\partial \widehat{N}}{\partial \widehat{c}}\, E\{c \,|\, I; \widehat{c}\}$$

$$= \widehat{N}\, [\widehat{c}\, \overline{p}\, g(\widehat{c})] - N\, \overline{p}\, g(\widehat{c})\, E\{c \,|\, I; \widehat{c}\} \stackrel{s}{=} \widehat{N}\, \widehat{c} - N\, E\{c \,|\, I; \widehat{c}\} = 0$$

$$\text{when } \widehat{c} = \frac{N\, E\{c \,|\, I; \widehat{c}\}}{\widehat{N}} = \frac{N\, E\{c \,|\, I; \widehat{c}\}}{N\, [G(\widehat{c}^*) + (1 - \overline{p})\, (1 - G(\widehat{c}^*))]}$$

$$= \frac{E\{c \,|\, I; \widehat{c}\}}{G(\widehat{c}^*) + [1 - \overline{p}]\, [1 - G(\widehat{c}^*)]} = \frac{E\{c \,|\, I; \widehat{c}\}}{1 - \overline{p}\, [1 - G(\widehat{c}^*)]}. \tag{45}$$

Conclusion 4. *If Assumptions 1 and 2 hold, then,* $\widehat{c}^*$ *is independent of* A*. Furthermore,* $\widehat{c}^* < E\{c\}$, $\frac{\partial}{\partial \overline{p}}\{\widehat{c}^*\} < 0$, *and* $\widehat{c}^* \to \underline{c}$ *as* $\overline{p} \to 1$.

Proof. (35) and Conclusion 3 imply that under an optimally designed OO policy, average surplus is:

$$W^* = -\frac{T}{N}\left[\frac{\int_{\underline{c}}^{\widehat{c}^*} c\, dG(c) + (1-\overline{p})\int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)}{G(\widehat{c}^*) + (1-\overline{p})(1-G(\widehat{c}^*))}\right] - \frac{A}{N} = -\frac{T}{N}\,\widehat{c}^* - \frac{A}{N}\,. \tag{46}$$

It is apparent from (34) that $\widehat{c}^*$ is independent of A. From (46), average surplus is $-\frac{T}{N}\widehat{c}^* - \frac{A}{N}$ under an optimal OO policy. Average surplus is $-\frac{T}{N}E\{c\}$ under mandatory jury service. Conclusion 2 implies that if $A = 0$, then all individuals are better off under the optimal OO policy. Therefore:

$$-\frac{T}{N}\,\widehat{c}^* > -\frac{T}{N}E\{c\} \quad \Leftrightarrow \quad \widehat{c}^* < E\{c\}\,.$$

From (34):

$$\widehat{c}^*[G(\widehat{c}^*) + (1-\overline{p})(1-G(\widehat{c}^*))] = \int_{\underline{c}}^{\widehat{c}^*} c\, dG(c) + [1-\overline{p}]\int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)\,. \tag{47}$$

Differentiating (47) provides:

$$\begin{aligned}
&\frac{\partial\widehat{c}^*}{\partial\overline{p}}[G(\widehat{c}^*) + (1-\overline{p})(1-G(\widehat{c}^*))] \\
&\qquad\qquad + \widehat{c}^*\left[g(\widehat{c}^*)\frac{\partial\widehat{c}^*}{\partial\overline{p}} - (1-\overline{p})\,g(\widehat{c}^*)\frac{\partial\widehat{c}^*}{\partial\overline{p}} - (1-G(\widehat{c}^*))\right] \\
&= \widehat{c}^*\,g(\widehat{c}^*)\frac{\partial\widehat{c}^*}{\partial\overline{p}} - [1-\overline{p}]\widehat{c}^*\,g(\widehat{c}^*)\frac{\partial\widehat{c}^*}{\partial\overline{p}} - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c) \\
&\Rightarrow \frac{\partial\widehat{c}^*}{\partial\overline{p}}[G(\widehat{c}^*) + (1-\overline{p})(1-G(\widehat{c}^*))] = \widehat{c}^*[1-G(\widehat{c}^*)] - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)
\end{aligned}$$

$$\Rightarrow \frac{\partial\widehat{c}^*}{\partial\overline{p}} = \frac{\widehat{c}^*[1-G(\widehat{c}^*)] - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)}{G(\widehat{c}^*) + [1-\overline{p}][1-G(\widehat{c}^*)]} < 0\,. \tag{48}$$

The inequality in (48) holds because: (i) $\widehat{c}^*[1-G(\widehat{c}^*)] - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c) = \int_{\widehat{c}^*}^{\overline{c}}[\widehat{c}^* - c]\, dG(c) < 0$; and (ii) $G(\widehat{c}^*) + [1-\overline{p}][1-G(\widehat{c}^*)] > 0$.

Finally, observe from (47) that if $\overline{p} \to 1$, then:

$$\widehat{c}^* G(\widehat{c}^*) \;\rightarrow\; \int_{\underline{c}}^{\widehat{c}^*} c\, dG(c) \;\;\Leftrightarrow\;\; \int_{\underline{c}}^{\widehat{c}^*} [\,\widehat{c}^* - c\,]\, dG(c) \;\rightarrow\; 0 \;\;\Leftrightarrow\;\; \widehat{c}^* \;\rightarrow\; \underline{c}\,. \quad \blacksquare$$

Explanation. $\widehat{c}^*$ will be interior because when $\widehat{c}$ is initially high (e.g., close to $\overline{c}$), the expected cost of a selected juror is relatively high and the size of the jury pool is relatively large. Consequently, welfare increases as $\widehat{c}$ declines, which reduces $E\{\,c\,|\,I\,;\,\widehat{c}\}$ relatively rapidly. (Observe from (43) that $E\{\,c\,|\,I\,;\,\widehat{c}\}$ is directly proportional to $\widehat{c}$.) In contrast, when $\widehat{c}$ is low (e.g., close to $\underline{c}$), the expected cost of an individual in the jury pool is relatively low and the size of the jury pool is relatively small. Consequently, welfare declines as $\widehat{c}$ declines, which reduces $E\{\,c\,|\,I\,;\,\widehat{c}\}$ at a relatively slow rate.

To understand why the optimal $\widehat{c}$ is less than $E\{c\}$, observe from (45) that:

$$E\{c\} \;>\; \widehat{c}^* \;\;\Leftrightarrow\;\; \frac{E\{c\}}{N} \;>\; \frac{E\{\,c\,|\,I\,;\,\widehat{c}\,\}}{N\,[\,1 - \overline{p}\,(\,1 - G(\widehat{c}^*\,)\,)\,]} \;=\; \frac{E\{\,c\,|\,I\,;\,\widehat{c}\,\}}{\widehat{N}^*}\,.$$

Therefore, $\widehat{c}^* < E\{c\}$ simply indicates that the optimal opt-out policy reduces the expected cost per individual of those in the jury pool.

The limiting case considered in Conclusion 4 may not be consistent with the maintained assumption that Assumption 2 holds (i.e., that the adequate jury pool constraint is always satisfied). If (nearly) all opt-out requests are satisfied (because $\overline{p} \rightarrow 1$), then welfare is maximized by allowing (nearly) all individuals to opt out of jury service, thereby avoiding (nearly) all costs of performing jury service. It seems that as $\overline{p} \rightarrow 1$ (so the "trial by peers" requirement is effectively not relevant), opting out will have to be limited (so $\widehat{c}^*$ does not approach $\underline{c}$) to ensure that the adequate jury pool constraint is satisfied.

From (4) and Conclusion 2, an individual prefers the OO policy to mandatory jury service if:

$$c \;>\; \widehat{c}^* + \frac{A}{T}\,\frac{\widehat{N}}{\overline{p}\,G\,(\widehat{c}^*)\;N} \quad \text{or} \quad c \;<\; \widehat{c}^* - \frac{A}{T}\,\frac{\widehat{N}}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]\,N}$$

$$\Leftrightarrow \quad c \;>\; \widehat{c}^* + \frac{A}{T}\,a_2 \quad \text{or} \quad c \;<\; \widehat{c}^* - \frac{A}{T}\,a_1$$

$$\text{where} \quad a_1 \;\equiv\; \frac{1 - \overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]} \quad \text{and} \quad a_2 \;\equiv\; \frac{1 - \overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]}{\overline{p}\;G\,(\widehat{c}^*)}\,. \tag{49}$$

(49) implies that the fraction of the population that prefers the OO policy to mandatory jury service is:

$$J^O(A) \;\equiv\; G\left(\widehat{c}^* - \frac{A}{T}\,a_1\right) + 1 - G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right)$$

whereas the fraction of the population that prefers mandatory jury service to the OO policy is:

$$J^M(A) \;\equiv\; 1 - \left[G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) + 1 - G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) \right]$$

$$= \; G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) - G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) .$$

Therefore, the difference between the fraction of individuals that prefer the optimal OO policy and the fraction that prefer mandatory jury service is:

$$J^O(A) - J^M(A) \;=\; G\left(\widehat{c}^* - \frac{A}{T}\,a_1\right) + 1 - G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) + G\left(\widehat{c}^* - \frac{A}{T}\,a_1\right) - G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right)$$

$$= \; 1 - 2\left[G\left(\widehat{c}^* + \frac{A}{T}\,a_2\right) - G\left(\widehat{c}^* - \frac{A}{T}\,a_1\right) \right] \;\equiv\; J\,(A) \,. \tag{50}$$

If $J\,(A) > 0$, then a majority of the population prefers the optimal OO policy. If $J\,(A) < 0$, then a majority of the population prefers the mandatory jury service policy.

Lemma 4. *If Assumptions 1 and 2 hold, then there exists a unique $A^* > 0$ such that: (i) $J\,(A) > 0$ for all $A < A^*$; (ii) $J\,(A) < 0$ for all $A > A^*$; and (iii) $J\,(A^*) = 0$.*

Proof. The conclusion holds because it is apparent from (50) that $J(A)$ is a decreasing function of A, $J(0) = 1$ and $J(A) \to -1$ as $A \to \infty$. ■

Lemma 4 implies that the majority of the population will support the OO policy if its administrative costs are sufficiently small, whereas a majority will oppose the OO policy if its administrative costs are sufficiently large.

From (46), average surplus is $-\frac{T}{N}\,\widehat{c}^* - \frac{A}{N}$ under the optimal OO policy. Average surplus is $-\frac{T}{N}\,E\{c\}$ when jury service is mandated. Therefore, aggregate surplus is greater under the optimal OO policy than under mandated jury service if and only if:

$$-\frac{T}{N}\,\widehat{c}^* - \frac{A}{N} \;>\; -\frac{T}{N}\,E\{c\} \quad\Leftrightarrow\quad \frac{T}{N}\,\widehat{c}^* + \frac{A}{N} \;<\; \frac{T}{N}\,E\{c\} \quad\Leftrightarrow\quad \widehat{c}^* + \frac{A}{T} \;<\; E\{c\} \,. \tag{51}$$

Define $\quad H(A) \;\equiv\; \widehat{c}^* + \dfrac{A}{T} - E\{c\} \,. \qquad (52)$

Lemma 5. *If Assumptions 1 and 2 hold, then there exists a unique $A^{**} > 0$, such that: (i) $H\,(A) < 0$ for all $A < A^{**}$; (ii) $H\,(A) > 0$ for all $A > A^{**}$; and (iii) $H\,(A^{**}) = 0$.*

Proof. It is apparent from (52) that $H'(A) > 0$ and $H(\infty) > 0$. Conclusion 4 implies $H(0) = \widehat{c}^* - E\{c\} < 0$. ■

Lemma 5 implies that adoption of the OO policy will increase expected welfare when A is sufficiently small, but reduce expected welfare when A is sufficiently large.

Lemmas 4 and 5 provide the following conclusions.

Lemma 6. *If Assumptions 1 and 2 hold and* $A^* < A^{**}$*, then:*

1. If $A < A^*$, then a majority of individuals prefer the optimal OO policy, which provides a higher level of aggregate surplus than mandatory jury service.
2. If $A \in (A^*, A^{**})$, then only a minority of individuals prefer the optimal OO policy even though it secures a higher level of aggregate surplus than mandatory jury service.
3. If $A > A^{**}$, then a majority of individuals prefer mandatory jury service, which secures a higher level of aggregate surplus than the OO policy.

Lemma 7. *If Assumptions 1 and 2 hold and* $A^* > A^{**}$*. Then:*

1. If $A < A^{**}$, then a majority of individuals prefer the optimal OO policy, which provides a higher level of aggregate surplus than mandatory jury service.
2. If $A \in (A^{**}, A^*)$, then a majority of individuals prefer the optimal OO policy, even though it secures a lower level of aggregate surplus than mandatory jury service.
3. If $A > A^*$, then a majority of individuals prefer mandatory jury service, which secures a higher level of aggregate surplus than an OO policy.

Lemmas 6 and 7 establish that for small or large administrative costs, a majority of the population prefers the surplus-maximizing policy. In contrast, for intermediate values of administrative costs, only a minority of the population prefers the surplus-maximizing policy. Formally:

Conclusion 5. *Suppose Assumptions 1 and 2 hold. Then a majority of the population prefers the surplus-maximizing policy if* $A < Min\{A^*, A^{**}\}$ *or* $A > Max\{A^*, A^{**}\}$*. Only a minority of the population prefers the surplus-maximizing policy if* $A \in (Min\{A^*, A^{**}\}, Max\{A^*, A^{**}\})$*.*

We cannot determine whether $A^* > A^{**}$ or $A^* < A^{**}$ in general. The relationship depends on the distribution $G(c)$.

Conclusion 6. *$A^* = A^{**}$ when $g(c) = \frac{1}{\overline{c}-\underline{c}}$ for all $c \in [\underline{c}, \overline{c}]$.*

Proof. From (34), when $g(c) = \frac{1}{\overline{c}-\underline{c}}$ for all $c \in [\underline{c}, \overline{c}]$:

$$\widehat{c}^*\,[G(\widehat{c}^*) + (1-\overline{p})(1-G(\widehat{c}^*))] \;=\; \int_{\underline{c}}^{\widehat{c}^*} c\, dG(c) + [1-\overline{p}] \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)$$

$$\Leftrightarrow \;\; \widehat{c}^* \left[\frac{\widehat{c}^* - \underline{c}}{\overline{c}-\underline{c}} + (1-\overline{p}) \left(1 - \frac{\widehat{c}^* - \underline{c}}{\overline{c}-\underline{c}} \right) \right] \;=\; \frac{c^2}{2\,[\overline{c}-\underline{c}]} \Bigg|_{\underline{c}}^{\widehat{c}^*} + [1-\overline{p}]\; \frac{c^2}{2\,[\overline{c}-\underline{c}]} \Bigg|_{\widehat{c}^*}^{\overline{c}}$$

$$= \frac{[\widehat{c}^* - \underline{c}]\,[\widehat{c}^* + \underline{c}]}{2\,[\overline{c}-\underline{c}]} + [1-\overline{p}]\, \frac{[\overline{c} - \widehat{c}^*]\,[\overline{c} + \widehat{c}^*]}{2\,[\overline{c}-\underline{c}]}$$

$$\Leftrightarrow \;\; \widehat{c}^* \left[\frac{\widehat{c}^* - \underline{c}}{\overline{c}-\underline{c}} + (1-\overline{p})\, \frac{\overline{c} - \widehat{c}^*}{\overline{c}-\underline{c}} \right] \;=\; \frac{[\widehat{c}^* - \underline{c}]\,[\widehat{c}^* + \underline{c}]}{2\,[\overline{c}-\underline{c}]} + [1-\overline{p}]\, \frac{[\overline{c} - \widehat{c}^*]\,[\overline{c} + \widehat{c}^*]}{2\,[\overline{c}-\underline{c}]}$$

$$\Leftrightarrow \;\; \widehat{c}^* \left[\frac{\widehat{c}^* - \underline{c}}{\overline{c}-\underline{c}} \right] - \frac{[\widehat{c}^* - \underline{c}]\,[\widehat{c}^* + \underline{c}]}{2\,[\overline{c}-\underline{c}]} \;=\; [1-\overline{p}]\, \frac{[\overline{c} - \widehat{c}^*]\,[\overline{c} + \widehat{c}^*]}{2\,[\overline{c}-\underline{c}]} - [1-\overline{p}]\, \widehat{c}^* \left[\frac{\overline{c} - \widehat{c}^*}{\overline{c}-\underline{c}} \right]$$

$$\Leftrightarrow \;\; [\widehat{c}^* - \underline{c}] \left[\frac{\widehat{c}^*}{\overline{c}-\underline{c}} - \frac{\widehat{c}^* + \underline{c}}{2\,[\overline{c}-\underline{c}]} \right] \;=\; [1-\overline{p}]\,[\overline{c} - \widehat{c}^*] \left[\frac{\overline{c} + \widehat{c}^*}{2\,[\overline{c}-\underline{c}]} - \frac{\widehat{c}^*}{\overline{c}-\underline{c}} \right]$$

$$\Leftrightarrow \;\; [\widehat{c}^* - \underline{c}]\; \frac{2\,\widehat{c}^* - [\widehat{c}^* + \underline{c}]}{2\,[\overline{c}-\underline{c}]} \;=\; [1-\overline{p}]\,[\overline{c} - \widehat{c}^*]\; \frac{\overline{c} + \widehat{c}^* - 2\,\widehat{c}^*}{2\,[\overline{c}-\underline{c}]}$$

$$\Leftrightarrow \;\; [\widehat{c}^* - \underline{c}]\; \frac{\widehat{c}^* - \underline{c}}{2\,[\overline{c}-\underline{c}]} \;=\; [1-\overline{p}]\,[\overline{c} - \widehat{c}^*]\; \frac{\overline{c} - \widehat{c}^*}{2\,[\overline{c}-\underline{c}]}$$

$$\Leftrightarrow \;\; [\widehat{c}^* - \underline{c}]^2 \;=\; [1-\overline{p}]\,[\overline{c} - \widehat{c}^*]^2 \;\;\Leftrightarrow\;\; \widehat{c}^* - \underline{c} \;=\; \sqrt{1-\overline{p}}\;[\overline{c} - \widehat{c}^*]$$

$$\Leftrightarrow \;\; \widehat{c}^* + \widehat{c}^* \sqrt{1-\overline{p}} \;=\; \overline{c}\sqrt{1-\overline{p}} + \underline{c} \;\;\Leftrightarrow\;\; \widehat{c}^* \left[1 + \sqrt{1-\overline{p}} \right] \;=\; \underline{c} + \overline{c}\sqrt{1-\overline{p}}$$

$$\Leftrightarrow \;\; \widehat{c}^* \;=\; \left[\frac{1}{1+\sqrt{1-\overline{p}}} \right] \underline{c} + \left[\frac{\sqrt{1-\overline{p}}}{1+\sqrt{1-\overline{p}}} \right] \overline{c}. \tag{53}$$

From (50), when $g(c) = \frac{1}{\overline{c}-\underline{c}}$ for all $c \in [\underline{c}, \overline{c}]$:

$$G\left(\widehat{c}^* + \frac{A^*}{T}\, a_2 \right) - G\left(\widehat{c}^* - \frac{A^*}{T}\, a_1 \right) \;=\; \frac{1}{2}$$

$$\Leftrightarrow \quad \frac{\widehat{c}^* + \frac{A^*}{T}\, a_2 - \underline{c}}{\overline{c} - \underline{c}} - \frac{\widehat{c}^* - \frac{A^*}{T}\, a_1 - \underline{c}}{\overline{c} - \underline{c}} \;=\; \frac{1}{2}$$

$$\Leftrightarrow \quad \left[\frac{1}{\overline{c} - \underline{c}}\right] \frac{A^*}{T} \, [\, a_1 + a_2\,] \;=\; \frac{1}{2}\,. \tag{54}$$

From (49):

$$a_1 + a_2 \;=\; \frac{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]} + \frac{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]}{\overline{p}\; G\,(\widehat{c}^*)}$$

$$= \frac{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]}{\overline{p}} \left[\frac{1}{1 - G\,(\widehat{c}^*)} + \frac{1}{G\,(\widehat{c}^*)}\right] = \frac{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]\, G\,(\widehat{c}^*)}\,. \tag{55}$$

(54) and (55) imply:

$$\left[\frac{1}{\overline{c} - \underline{c}}\right] \frac{A^*}{T} \; \frac{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]\, G\,(\widehat{c}^*)} \;=\; \frac{1}{2}$$

$$\Leftrightarrow \quad \frac{A^*}{T} \;=\; \left[\frac{\overline{c} - \underline{c}}{2}\right] \frac{\overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]\, G\,(\widehat{c}^*)}{1 - \overline{p}\,[\, 1 - G\,(\widehat{c}^*)\,]} \;=\; \frac{\overline{c} - \underline{c}}{2} \left[\frac{\overline{p}\left[1 - \frac{\widehat{c}^* - \underline{c}}{\overline{c} - \underline{c}}\right] \frac{\widehat{c}^* - \underline{c}}{\overline{c} - \underline{c}}}{1 - \overline{p}\left[1 - \frac{\widehat{c}^* - \underline{c}}{\overline{c} - \underline{c}}\right]}\right]$$

$$= \frac{\overline{c} - \underline{c}}{2} \left[\frac{\overline{p}\left[\frac{\overline{c} - \widehat{c}^*}{\overline{c} - \underline{c}}\right] \frac{\widehat{c}^* - \underline{c}}{\overline{c} - \underline{c}}}{1 - \overline{p}\left[\frac{\overline{c} - \widehat{c}^*}{\overline{c} - \underline{c}}\right]}\right] \;=\; \frac{1}{2}\left[\frac{\overline{p}\,[\,\overline{c} - \widehat{c}^*\,]\,[\,\widehat{c}^* - \underline{c}\,]}{\overline{c} - \underline{c} - \overline{p}\,[\,\overline{c} - \widehat{c}^*\,]}\right]. \tag{56}$$

From (53):

$$\overline{c} - \widehat{c}^* \;=\; \overline{c} - \left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] \underline{c} - \left[\frac{\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\right] \overline{c}$$

$$= \left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] \overline{c} - \left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] \underline{c} \;=\; \frac{\overline{c} - \underline{c}}{1 + \sqrt{1 - \overline{p}}}\,. \tag{57}$$

Also:

$$\widehat{c}^* - \underline{c} \;=\; \left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] \underline{c} + \left[\frac{\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\right] \overline{c} - \underline{c}$$

$$= \left[\frac{\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\right] \overline{c} - \left[\frac{\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\right] \underline{c} \;=\; \frac{[\,\overline{c} - \underline{c}\,]\,\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\,. \tag{58}$$

(56), (57), and (58) imply:

$$\frac{A^*}{T} \;=\; \frac{1}{2}\left(\frac{\overline{p}\,[\,\overline{c} - \widehat{c}^*\,]\,[\,\widehat{c}^* - \underline{c}]}{\overline{c} - \underline{c} - \overline{p}\,[\,\overline{c} - \widehat{c}^*\,]}\right) \;=\; \frac{1}{2}\left(\frac{\overline{p}\left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] \frac{\sqrt{1 - \overline{p}}}{1 + \sqrt{1 - \overline{p}}}\,[\,\overline{c} - \underline{c}\,]^2}{\overline{c} - \underline{c} - \overline{p}\left[\frac{1}{1 + \sqrt{1 - \overline{p}}}\right] [\,\overline{c} - \underline{c}\,]}\right)$$

$$= \frac{1}{2}\left(\frac{\overline{p}\left[\frac{1}{1+\sqrt{1-\overline{p}}}\right]\left[\frac{\sqrt{1-\overline{p}}}{1+\sqrt{1-\overline{p}}}\right][\,\overline{c}-\underline{c}\,]}{1-\overline{p}\left[\frac{1}{1+\sqrt{1-\overline{p}}}\right]}\right) = \frac{1}{2}\left(\frac{\overline{p}\left[\frac{\sqrt{1-\overline{p}}}{1+\sqrt{1-\overline{p}}}\right][\,\overline{c}-\underline{c}\,]}{1-\overline{p}+\sqrt{1-\overline{p}}}\right)$$

$$= \frac{1}{2}\left(\frac{\overline{p}\left[\frac{\sqrt{1-\overline{p}}}{1+\sqrt{1-\overline{p}}}\right][\,\overline{c}-\underline{c}\,]}{\sqrt{1-\overline{p}}\,\left[\,1+\sqrt{1-\overline{p}}\,\right]}\right) = \frac{\overline{p}\,[\,\overline{c}-\underline{c}\,]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]^2}\,. \tag{59}$$

(52) and (53) imply:

$$\frac{A^{**}}{T} = E\{c\}-\widehat{c}^{\,*} = \frac{\underline{c}+\overline{c}}{2}-\left[\frac{1}{1+\sqrt{1-\overline{p}}}\right]\underline{c}-\left[\frac{\sqrt{1-\overline{p}}}{1+\sqrt{1-\overline{p}}}\right]\overline{c}$$

$$= \frac{[\,\underline{c}+\overline{c}\,]\left[\,1+\sqrt{1-\overline{p}}\,\right]-2\,\underline{c}-2\,\overline{c}\,\sqrt{1-\overline{p}}}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]}$$

$$= \frac{\overline{c}\left[\,1+\sqrt{1-\overline{p}}-2\sqrt{1-\overline{p}}\,\right]+\underline{c}\left[\,1+\sqrt{1-\overline{p}}-2\,\right]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]}$$

$$= \frac{\overline{c}\left[\,1-\sqrt{1-\overline{p}}\,\right]-\underline{c}\left[\,1-\sqrt{1-\overline{p}}\,\right]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]} = \frac{\left[\,1-\sqrt{1-\overline{p}}\,\right][\,\overline{c}-\underline{c}\,]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]}$$

$$= \frac{\left[\,1-\sqrt{1-\overline{p}}\,\right][\,\overline{c}-\underline{c}\,]\left[\,1+\sqrt{1-\overline{p}}\,\right]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]^2} = \frac{\overline{p}\,[\,\overline{c}-\underline{c}\,]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]^2}\,. \tag{60}$$

(59) and (60) imply:

$$\frac{A^{*}}{T} = \frac{\overline{p}\,[\,\overline{c}-\underline{c}\,]}{2\left[\,1+\sqrt{1-\overline{p}}\,\right]^2} = \frac{A^{**}}{T} \;\Rightarrow\; A^{*} = A^{**}. \quad \blacksquare$$

Piecewise Linear, Inverted V-Shaped Density

Suppose $g(c)$ is piecewise linear, symmetric around its mean, increasing for $c \leq E(c)$, and decreasing for $c \geq E(c)$. Formally, for $g(c)$ on $[0, 2L]$, $L > 0$:

$$g(c) = \begin{cases} \frac{c}{L^2} & if \quad 0 \leq c \leq L \\ \frac{2}{L} - \frac{c}{L^2} & if \quad L \leq c \leq 2L. \end{cases} \tag{61}$$

Conclusion 7. *$A^{**} > A^*$ if (61) holds and $\overline{p} \in (0,1)$.*

Proof. When (61) holds, the numerator in the expression for $\widehat{c}^* < L$ in (34) is:

$$\int_0^{\widehat{c}^*} c\, dG(c) + [1-\overline{p}] \left[\int_{\widehat{c}^*}^{L} c\, dG(c) + \int_L^{2L} c\, dG(c) \right]$$

$$= \int_0^{\widehat{c}^*} \frac{c^2}{L^2}\, dc + [1-\overline{p}] \left[\int_{\widehat{c}^*}^{L} \frac{c^2}{L^2}\, dc + \int_L^{2L} c \left(\frac{2}{L} - \frac{c}{L^2} \right) dc \right]$$

$$= \left[\frac{c^3}{3L^2} \right]_0^{\widehat{c}^*} + [1-\overline{p}] \left[\left(\frac{c^3}{3L^2} \right)_{\widehat{c}^*}^{L} + \left(\frac{c^2}{L} \right)_L^{2L} - \left(\frac{c^3}{3L^2} \right)_L^{2L} \right]$$

$$= \frac{(\widehat{c}^*)^3}{3L^2} + [1-\overline{p}] \left[\frac{L}{3} - \frac{(\widehat{c}^*)^3}{3L^2} + 3L - \frac{7L}{3} \right] = \frac{(\widehat{c}^*)^3}{3L^2} + [1-\overline{p}] \left[L - \frac{(\widehat{c}^*)^3}{3L^2} \right]. \tag{62}$$

When (61) holds, the denominator in the expression for $\widehat{c}^*$ in (34) is:

$$G(\widehat{c}^*) + [1-\overline{p}][1 - G(\widehat{c}^*)] = \int_0^{\widehat{c}^*} \frac{c}{L^2}\, dc + [1-\overline{p}] \left[1 - \int_0^{\widehat{c}^*} \frac{c}{L^2}\, dc \right]$$

$$= \frac{(\widehat{c}^*)^2}{2L^2} + [1-\overline{p}] \left[1 - \frac{(\widehat{c}^*)^2}{2L^2} \right]. \tag{63}$$

(34), (62), and (63) imply that when (61) holds:

$$\widehat{c}^* \left[\frac{(\widehat{c}^*)^2}{2L^2} + [1-\overline{p}] \left(1 - \frac{(\widehat{c}^*)^2}{2L^2} \right) \right] = \frac{(\widehat{c}^*)^3}{3L^2} + [1-\overline{p}] \left[L - \frac{(\widehat{c}^*)^3}{3L^2} \right]$$

$$\Rightarrow \frac{(\widehat{c}^*)^3}{2L^2} - \frac{(\widehat{c}^*)^3}{3L^2} = [1-\overline{p}] \left[L - \frac{(\widehat{c}^*)^3}{3L^2} - \widehat{c}^* + \frac{(\widehat{c}^*)^3}{2L^2} \right]$$

$$\Rightarrow \quad \frac{(\widehat{c}^*)^3}{6\,L^2} - [\,1-\overline{p}\,]\left[L + \frac{(\widehat{c}^*)^3}{6\,L^2} - \widehat{c}^*\right] \;=\; 0$$

$$\Rightarrow \quad \frac{\overline{p}\;(\widehat{c}^*)^3}{6\,L^2} - [\,1-\overline{p}\,][\,L-\widehat{c}^*\,] \;=\; 0 \;\;\Rightarrow\;\; \overline{p}\;(\widehat{c}^*)^3 \;=\; 6\,L^2\,[\,1-\overline{p}\,][\,L-\widehat{c}^*\,]\,. \tag{64}$$

From (50), A^* is defined by:

$$1 - 2\left[G\left(\widehat{c}^* + \frac{A^*}{T}\,a_2\right) - G\left(\widehat{c}^* - \frac{A^*}{T}\,a_1\right)\right] \;=\; 0\,. \tag{65}$$

Observe that:

$$A^* \;<\; A^{**} \quad\Leftrightarrow\quad 1 - 2\left[G\left(\widehat{c}^* + \frac{A^{**}}{T}\,a_2\right) - G\left(\widehat{c}^* - \frac{A^{**}}{T}\,a_1\right)\right] \;<\; 0$$

$$\Leftrightarrow\quad G\left(\widehat{c}^* + \frac{A^{**}}{T}\,a_2\right) - G\left(\widehat{c}^* - \frac{A^{**}}{T}\,a_1\right) \;>\; \frac{1}{2}\,. \tag{66}$$

The first equivalence in (66) holds because the last inequality states that more than half of the population prefers mandatory jury service to optional jury service when $A = A^{**}$. By definition, the same number of individuals prefer mandatory jury service and optional jury service if $A = A^*$. Therefore, A^{**} must exceed A^* and so for $A \in (A^*, A^{**})$, the majority will favor mandatory jury service even though welfare would be higher under optional jury service.

Because $\frac{A^{**}}{T} = E\{c\} - \widehat{c}^*$ from (52) and $a_2 \equiv \frac{1-\overline{p}\,[1-G(\widehat{c}^*)]}{\overline{p}\,G(\widehat{c}^*)}$ from (49):

$$\begin{aligned}
\widehat{c}^* + \frac{A^{**}}{T}\,a_2 \;&=\; \widehat{c}^* + [\,E\{c\} - \widehat{c}^*\,]\left[\frac{1-\overline{p}\,[\,1-G(\widehat{c}^*)\,]}{\overline{p}\;G(\widehat{c}^*)}\right] \\
&=\; \widehat{c}^* + [\,E\{c\} - \widehat{c}^*\,]\left[1 + \frac{1-\overline{p}}{\overline{p}\;G(\widehat{c}^*)}\right] \\
&=\; \widehat{c}^* + E\{c\} - \widehat{c}^* + [\,E\{c\} - \widehat{c}^*\,]\left[\frac{1-\overline{p}}{\overline{p}\;G(\widehat{c}^*)}\right] \\
&=\; E\{c\} + [\,E\{c\} - \widehat{c}^*\,]\left[\frac{1-\overline{p}}{\overline{p}\;G(\widehat{c}^*)}\right].
\end{aligned} \tag{67}$$

Because $\frac{A^{**}}{T} = E\{c\} - \widehat{c}^*$ from (52) and $a_1 \equiv \frac{1-\overline{p}\,[1-G(\widehat{c}^*)]}{\overline{p}\,[1-G(\widehat{c}^*)]}$ from (49):

$$\begin{aligned}
\widehat{c}^* - \frac{A^{**}}{T}\,a_1 \;&=\; \widehat{c}^* - [\,E\{c\} - \widehat{c}^*\,]\;\frac{1-\overline{p}\,[\,1-G(\widehat{c}^*)\,]}{\overline{p}\,[\,1-G(\widehat{c}^*)\,]} \\
&=\; \widehat{c}^* - [\,E\{c\} - \widehat{c}^*\,]\left[\frac{1}{\overline{p}\,[\,1-G(\widehat{c}^*)\,]} - 1\right]
\end{aligned}$$

$$= \widehat{c}^* + E\{c\} - \widehat{c}^* - [\, E\{c\} - \widehat{c}^* \,] \; \frac{1}{\overline{p}\,[\, 1 - G(\widehat{c}^*) \,]}$$

$$= E\{c\} - [\, E\{c\} - \widehat{c}^* \,] \; \frac{1}{\overline{p}\,[\, 1 - G(\widehat{c}^*) \,]} \, . \tag{68}$$

(66), (67), and (68) imply:

$$A^* \; < \; A^{**} \;\; \text{if} \;\; G(E\{c\} + [\, E\{c\} - \widehat{c}^* \,]\, \alpha_2) - \; G(E\{c\} - [\, E\{c\} - \widehat{c}^* \,]\, \alpha_1) \;\; > \; \frac{1}{2} \tag{69}$$

$$\text{where} \quad \alpha_1 \;\equiv\; \frac{1}{\overline{p}\,[\, 1 - G(\widehat{c}^*) \,]} \quad \text{and} \quad \alpha_2 \;\equiv\; \frac{1 - \overline{p}}{\overline{p}\; G(\widehat{c}^*)} \, . \tag{70}$$

The left hand side of the second inequality in (69) is the area under $g(c)$ for c between $E\{c\} - [\, E\{c\} - \widehat{c}^* \,]\, \alpha_1$ and $E\{c\} + [\, E\{c\} - \widehat{c}^* \,]\, \alpha_2$. This area is the sum of the areas under $g(c)$ for c between: (i) $E\{c\} - [\, E\{c\} - \widehat{c}^* \,]\, \alpha_1$ and L; and (ii) L and $E\{c\} + [\, E\{c\} - \widehat{c}^* \,]\, \alpha_2$.

From (61), the area under $g(c)$ for c between $E\{c\} - [\, E\{c\} - \widehat{c}^* \,]\, \alpha_1$ and L is:

$$\int_{L - [\, L - \widehat{c}^*]\alpha_1}^{L} \frac{c}{L^2}\, dc \; = \; \frac{c^2}{2L^2} \bigg|_{L - [\, L - \widehat{c}^*]\alpha_1}^{L} \; = \; \frac{1}{2} - \frac{1}{2\,L^2} \left[\, L - (L - \widehat{c}^*)\, \alpha_1 \,\right]^2 . \tag{71}$$

From (61), the area under $g(c)$ for c between L and $E\{c\} + [\, E\{c\} - \widehat{c}^* \,]\, \alpha_2$ is:

$$\int_{L}^{L + [\, L - \widehat{c}^*]\alpha_2} \left[\frac{2}{L} - \frac{c}{L^2} \right] dc \; = \; \frac{2\,c}{L} \bigg|_{L}^{L + [\, L - \widehat{c}^*]\alpha_2} - \; \frac{c^2}{2\,L^2} \bigg|_{L}^{L + [\, L - \widehat{c}^*]\alpha_2}$$

$$= \; \frac{2}{L} \, [\, L - \widehat{c}^* \,]\, \alpha_2 - \frac{1}{2\,L^2} \, [\, L + (\, L - \widehat{c}^* \,)\, \alpha_2 \,]^2 + \frac{1}{2}$$

$$= \; \frac{2}{L} \, [\, 1 - \widehat{c}^* \,]\, \alpha_2 - \frac{1}{2\,L^2} \left[\, L^2 + 2L\,(\, L - \widehat{c}^* \,)\, \alpha_2 + (\, L - \widehat{c}^*)^2\, (\alpha_2)^2 \,\right] + \frac{1}{2}$$

$$= \; \frac{1}{2} - \frac{1}{2\,L^2} \left[\, L^2 - 2\,L\,(\, 1 - \widehat{c}^* \,)\, \alpha_2 + (\, L - \widehat{c}^* \,)^2\, (\alpha_2)^2 \,\right]$$

$$= \; \frac{1}{2} - \frac{1}{2\,L^2} \left[\, L - (L - \widehat{c}^*)\, \alpha_2 \,\right]^2 . \tag{72}$$

(69), (71), and (72) imply:

$$A^* \; < \; A^{**} \;\; \Leftrightarrow \;\; 1 - \frac{1}{2\,L^2} \left[\, E\{c\} - (E\{c\} - \widehat{c}^*)\, \alpha_1 \,\right]^2 - \frac{1}{2\,L^2} \left[\, E\{c\} - (E\{c\} - \widehat{c}^*)\, \alpha_2 \,\right]^2 \;\; > \; \frac{1}{2}$$

$$\Leftrightarrow \;\; \left[\, E\{c\} - (E\{c\} - \widehat{c}^*)\, \alpha_1 \,\right]^2 + \left[\, E\{c\} - (E\{c\} - \widehat{c}^*)\, \alpha_2 \,\right]^2 \;\; < \;\; L^2$$

$$\Leftrightarrow \;\; \left[\, L - (L - \widehat{c}^*)\, \alpha_1 \,\right]^2 + \left[\, L - (L - \widehat{c}^*)\, \alpha_2 \,\right]^2 \;\; < \;\; L^2$$

$$\Leftrightarrow \;\; \left[\, L^2 - 2\,L\,(L - \widehat{c}^*)\, \alpha_1 + (L - \widehat{c}^*)^2\, \alpha_1^2 \,\right] + \left[\, L^2 - 2\,L\,(L - \widehat{c}^*)\, \alpha_2 + (L - \widehat{c}^*)^2\, \alpha_2^2 \,\right] \;\; < \;\; L^2$$

$$\Leftrightarrow \quad 2\,L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2 \left[\,\alpha_1^2 + \alpha_2^2\,\right] \;<\; L^2$$

$$\Leftrightarrow \quad L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2 \left[\,\alpha_1^2 + \alpha_2^2\,\right] \;<\; 0\,. \tag{73}$$

Observe that:

$$\begin{aligned}
& L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2 \left[\,\alpha_1^2 + \alpha_2^2\,\right] \\
&\quad = L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2 \left[\,\alpha_1^2 + \alpha_2^2 + 2\,\alpha_1\,\alpha_2 - 2\,\alpha_1\,\alpha_2\,\right] \\
&\quad = L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2 \left[\,(\alpha_1 + \alpha_2)^2 - 2\,\alpha_1\,\alpha_2\,\right] \\
&\quad = L^2 - 2\,L\,[\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] + [\,L - \widehat{c}^*\,]^2\,[\,\alpha_1 + \alpha_2\,]^2 - 2\,[\,L - \widehat{c}^*\,]^2\,\alpha_1\,\alpha_2 \\
&\quad = [\,L - (L - \widehat{c}^*)\,(\alpha_1 + \alpha_2)\,]^2 - 2\,[\,L - \widehat{c}^*\,]^2\,\alpha_1\,\alpha_2\,.
\end{aligned} \tag{74}$$

(73) and (74) imply:

$$A^* \;<\; A^{**} \quad \Leftrightarrow \quad [\,L - (L - \widehat{c}^*)\,(\alpha_1 + \alpha_2)\,]^2 \;<\; 2\,[\,L - \widehat{c}^*\,]^2\,\alpha_1\,\alpha_2$$

$$\Leftrightarrow \quad L - [\,L - \widehat{c}^*\,]\,[\,\alpha_1 + \alpha_2\,] < \sqrt{2}\,[\,L - \widehat{c}^*\,]\,\sqrt{\alpha_1\,\alpha_2}\,. \tag{75}$$

(70) implies:

$$2\,[\,L - \widehat{c}^*\,]^2\,\alpha_1\,\alpha_2 \;=\; 2\,[\,L - \widehat{c}^*\,]^2\;\frac{1}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]}\;\frac{1 - \overline{p}}{\overline{p}\;G\,(\widehat{c}^*)}$$

$$=\; 2\,[\,L - \widehat{c}^*\,]^2\;\frac{1 - \overline{p}}{(\overline{p})^2}\;\frac{1}{[\,1 - G\,(\widehat{c}^*)\,]\,G\,(\widehat{c}^*)} \;=\; 2\,[\,L - \widehat{c}^*\,]^2\;\frac{1 - \overline{p}}{(\overline{p})^2}\;\frac{1}{\left[\,1 - \frac{(\widehat{c}^*)^2}{2\,L^2}\,\right]\frac{(\widehat{c}^*)^2}{2\,L^2}}$$

$$=\; 2\,[\,L - \widehat{c}^*\,]^2\;\frac{1 - \overline{p}}{(\overline{p})^2}\;\frac{4\,L^4}{\left[\,2\,L^2 - (\widehat{c}^*)^2\,\right](\widehat{c}^*)^2}$$

$$\Rightarrow \quad \sqrt{2}\,[\,L - \widehat{c}^*\,]\,\sqrt{\alpha_1\,\alpha_2} \;=\; 2\,L^2\,\sqrt{2}\;\frac{[\,L - \widehat{c}^*\,]}{\widehat{c}^*}\;\frac{\sqrt{1 - \overline{p}}}{\overline{p}}\;\frac{1}{\sqrt{2\,L^2 - (\widehat{c}^*)^2}}\,. \tag{76}$$

(70) also implies:

$$\alpha_1 + \alpha_2 \;=\; \frac{1}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]} + \frac{1 - \overline{p}}{\overline{p}\;G\,(\widehat{c}^*)} \;=\; \frac{G\,(\widehat{c}^*) + [\,1 - \overline{p}\,]\,[\,1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]\,G\,(\widehat{c}^*)}$$

$$=\; \frac{G\,(\widehat{c}^*) + 1 - \overline{p} - G\,(\widehat{c}^*) + \overline{p}\;G\,(\widehat{c}^*)}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]\,G\,(\widehat{c}^*)} \;=\; \frac{1 - \overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]}{\overline{p}\,[\,1 - G\,(\widehat{c}^*)\,]\,G\,(\widehat{c}^*)}$$

$$=\; \frac{1 - \overline{p}\left[\,1 - \frac{(\widehat{c}^*)^2}{2\,L^2}\,\right]}{\overline{p}\left[\,1 - \frac{(\widehat{c}^*)^2}{2\,L^2}\,\right]\frac{(\widehat{c}^*)^2}{2\,L^2}} \;=\; \frac{2\,L^2\left[\,2\,L^2 - \overline{p}\left(2\,L^2 - (\widehat{c}^*)^2\right)\,\right]}{\overline{p}\left[\,2\,L^2 - (\widehat{c}^*)^2\,\right](\widehat{c}^*)^2}$$

$$\Rightarrow \;\; L - [\, L - \widehat{c}^{\,*} \,]\, [\, \alpha_1 + \alpha_2 \,] \;\; = \;\; L - [\, L - \widehat{c}^{\,*} \,] \; \frac{2\, L^2 \left[\, 2\, L^2 - \overline{p} \left(2\, L^2 - (\widehat{c}^{\,*})^2 \right) \right]}{\overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] (\widehat{c}^{\,*})^2} \,. \tag{77}$$

(75), (76), and (77) imply:

$$A^* \; < \; A^{**} \;\; \Leftrightarrow \;\; L - [\, L - \widehat{c}^{\,*} \,] \, \frac{2\, L^2 \left[\, 2\, L^2 - \overline{p} \left(2\, L^2 - (\widehat{c}^{\,*})^2 \right) \right]}{\overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] (\widehat{c}^{\,*})^2} \;\; < \;\; \frac{2\, L^2 \sqrt{2} \, [\, L - \widehat{c}^{\,*} \,] \sqrt{1 - \overline{p}}}{\widehat{c}^{\,*} \, \overline{p} \sqrt{2\, L^2 - (\widehat{c}^{\,*})^2}}$$

$$\begin{aligned} \Leftrightarrow \;\; & L \, \overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] (\widehat{c}^{\,*})^2 - 2\, L^2 \, [\, L - \widehat{c}^{\,*} \,] \left[\, 2\, L^2 - \overline{p} \left(2\, L^2 - (\widehat{c}^{\,*})^2 \right) \right] \\ & < \; \frac{\overline{p} \, (\widehat{c}^{\,*})^2 \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right]}{\overline{p} \, \widehat{c}^{\,*} \sqrt{2\, L^2 - (\widehat{c}^{\,*})^2}} \; 2\, L^2 \sqrt{2} \, [\, L - \widehat{c}^{\,*} \,] \sqrt{1 - \overline{p}} \\ & = \; 2\, L^2 \sqrt{2} \sqrt{1 - \overline{p}} \;\; [\, \widehat{c}^{\,*} \,] \, [\, L - \widehat{c}^{\,*} \,] \sqrt{2\, L^2 - (\widehat{c}^{\,*})^2} \,. \end{aligned} \tag{78}$$

Observe that:

$$\begin{aligned} & L \, \overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] (\widehat{c}^{\,*})^2 - 2\, L^2 \, [\, L - \widehat{c}^{\,*} \,] \left[\, 2\, L^2 - \overline{p} \left(2\, L^2 - (\widehat{c}^{\,*})^2 \right) \right] \\ & = L \, \overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] (\widehat{c}^{\,*})^2 - 4\, L^4 \, [\, L - \widehat{c}^{\,*} \,] + 2\, L^2 \, [\, L - \widehat{c}^{\,*} \,] \, \overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \,\right] \\ & = \; L \, \overline{p} \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 \right] (\widehat{c}^{\,*})^2 - 4\, L^4 \, [\, L - \widehat{c}^{\,*} \,] + 4\, L^4 \, \overline{p} \, [\, L - \widehat{c}^{\,*} \,] - 2\, L^2 \, \overline{p} \, [\, L - \widehat{c}^{\,*} \,] \, (\widehat{c}^{\,*})^2 \\ & = \; L \, \overline{p} \, (\widehat{c}^{\,*})^2 \left[\, 2\, L^2 - (\widehat{c}^{\,*})^2 - 2\, L \, (L - \widehat{c}^{\,*}) \right] - 4\, L^4 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \\ & = \; L \, \overline{p} \, (\widehat{c}^{\,*})^3 \, [\, 2\, L - \widehat{c}^{\,*} \,] - 4\, L^4 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \\ & = \; 6\, L^3 \, [\, 1 - \overline{p} \,] \, [\, L - \widehat{c}^{\,*} \,] \, [\, 2\, L - \widehat{c}^{\,*} \,] - 4\, L^4 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \qquad (79) \\ & = \; 2\, L^3 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \, [\, 3 \, (2\, L - \widehat{c}^{\,*}) - 2\, L \,] \\ & = \; 2\, L^3 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \, [\, 4\, L - 3\, \widehat{c}^{\,*} \,] \,. \qquad (80) \end{aligned}$$

The equality in (79) follows from (64). (78) and (80) imply:

$$\begin{aligned} A^* \; < \; A^{**} \;\; \Leftrightarrow \;\; & 2\, L^3 \, [\, L - \widehat{c}^{\,*} \,] \, [\, 1 - \overline{p} \,] \, [\, 4\, L - 3\, \widehat{c}^{\,*} \,] \\ & < \; 2\, L^2 \sqrt{2} \sqrt{1 - \overline{p}} \;\; [\, \widehat{c}^{\,*} \,] \, [\, L - \widehat{c}^{\,*} \,] \sqrt{2\, L^2 - (\widehat{c}^{\,*})^2} \,. \end{aligned} \tag{81}$$

From (64):

$$\overline{p} \; (\widehat{c}^{\,*})^3 - 6\, L^2 \, [\, 1 - \overline{p} \,] \, [\, L - \widehat{c}^{\,*} \,] \;\; = \;\; 0 \;\; \Rightarrow \;\; \overline{p} \; (\widehat{c}^{\,*})^3 + 6\, L^2 \, \overline{p} \, [\, L - \widehat{c}^{\,*} \,] - 6\, L^2 \, [\, L - \widehat{c}^{\,*} \,] \;\; = \;\; 0$$

$$\Rightarrow \;\; \overline{p} \left[\, (\widehat{c}^{\,*})^3 + 6\, L^2 \, (L - \widehat{c}^{\,*}) \right] \;\; = \;\; 6\, L^2 \, [\, L - \widehat{c}^{\,*} \,]$$

$$\Rightarrow \;\; \overline{p} \;\; = \;\; \frac{6\, L^2 \, [\, L - \widehat{c}^{\,*} \,]}{(\widehat{c}^{\,*})^3 + 6\, L^2 \, [\, L - \widehat{c}^{\,*} \,]} \;\; \Rightarrow \;\; 1 - \overline{p} \;\; = \;\; \frac{(\widehat{c}^{\,*})^3}{(\widehat{c}^{\,*})^3 + 6\, L^2 \, [\, L - \widehat{c}^{\,*} \,]} \,. \tag{82}$$

(82) implies:

$$\frac{d\,\overline{p}}{d\,\widehat{c}^*} \stackrel{s}{=} -\left[\,(\widehat{c}^*)^3 + 6L^2\,(L - \widehat{c}^*)\,\right] - [\,L - \widehat{c}^*\,]\left[\,3\,(\widehat{c}^*)^2 - 6\,L^2\,\right]$$

$$= -\,(\widehat{c}^*)^3 - 3\,[\,L - \widehat{c}^*\,]\,(\widehat{c}^*)^2 \;<\; 0 \quad \text{for } \widehat{c}^* \in (\,0, L\,];$$

$$\widehat{c}^* \;\to\; L \text{ as } \overline{p} \;\to\; 0\,; \text{ and } \widehat{c}^* \;\to\; 0 \text{ as } \overline{p} \;\to\; 1\,. \tag{83}$$

(82) and (83) imply that $\widehat{c}^* \in (0, L)$ if $\overline{p} \in (0, 1)$. Consequently, (81) implies that for $\overline{p} \in (0, 1)$:

$$A^* \;<\; A^{**} \;\;\Leftrightarrow\;\; L\,[\,1 - \overline{p}\,]\,[\,4\,L - 3\,\widehat{c}^*\,] \;<\; \sqrt{2}\,\sqrt{1 - \overline{p}}\;[\,\widehat{c}^*\,]\,\sqrt{2\,L^2 - (\widehat{c}^*)^2}$$

$$\Leftrightarrow\;\; L\,\sqrt{1 - \overline{p}}\;[\,4\,L - 3\,\widehat{c}^*\,] \;<\; \sqrt{2}\;[\,\widehat{c}^*\,]\,\sqrt{2\,L^2 - (\widehat{c}^*)^2}$$

$$\Rightarrow\;\; A^* \;<\; A^{**} \text{ if } f\,(\widehat{c}^*) \;\equiv\; 2\,(\widehat{c}^*)^2\left[\,2\,L^2 - (\widehat{c}^*)^2\right] - L^2\,[\,1 - \overline{p}\,]\,[\,4\,L - 3\,\widehat{c}^*\,]^2 \;>\; 0\,. \tag{84}$$

(82) and (84) imply that for $\overline{p} \in (0, 1)$:

$$f\,(\widehat{c}^*) \;=\; 2\,(\widehat{c}^*)^2\left[\,2\,L^2 - (\widehat{c}^*)^2\,\right] - \frac{L^2\,(\widehat{c}^*)^3\,[\,4\,L - 3\,\widehat{c}^*\,]^2}{(\widehat{c}^*)^3 + 6\,L^2\,[\,L - \widehat{c}^*\,]} \;>\; 0$$

$$\Leftrightarrow\;\; \eta\,(\widehat{c}^*) \;\equiv\; 2\left[\,(\widehat{c}^*)^3 + 6\,L^2\,(L - \widehat{c}^*)\,\right]\left[\,2\,L^2 - (\widehat{c}^*)^2\,\right] - L^2\,\widehat{c}^*\,[\,4\,L - 3\,\widehat{c}^*\,]^2 \;>\; 0\,. \tag{85}$$

Observe that:

$$\eta\,(\widehat{c}^*) \;=\; 2\left[\,2\,L^2\,(\widehat{c}^*)^3 + 12\,L^4\,(L - \widehat{c}^*) - (\widehat{c}^*)^5 - 6\,L^2\,(L - \widehat{c}^*)\,(\widehat{c}^*)^2\,\right]$$

$$-\; L^2\,\widehat{c}^*\left[\,16\,L^2 - 24\,L\,\widehat{c}^* + 9\,(\widehat{c}^*)^2\,\right]$$

$$=\; 4\,L^2\,(\widehat{c}^*)^3 + 24\,L^5 - 24\,L^4\,\widehat{c}^* \;-2\,(\widehat{c}^*)^5 - 12\,L^3\,(\,\widehat{c}^*)^2 + 12\,L^2\,(\widehat{c}^*)^3$$

$$-\; 16\,L^4\,\widehat{c}^* + 24\,L^3\,(\,\widehat{c}^*)^2 - 9\,L^2\,(\widehat{c}^*)^3$$

$$=\; 24\,L^5 - 40\,L^4\,\widehat{c}^* + 12\,L^3\,(\widehat{c}^*)^2 + 7\,L^2\,(\widehat{c}^*)^3 - 2\,(\widehat{c}^*)^5\,. \tag{86}$$

Differentiating (86) provides:

$$\eta'\,(\widehat{c}^*) \;=\; -\,40\,L^4 + 24\,L^3\,\widehat{c}^* + 21\,L^2\,(\widehat{c}^*)^2 - 10\,(\widehat{c}^*)^4$$

$$\Rightarrow\;\; \eta''\,(\widehat{c}^*) \;=\; 24\,L^3 + 42\,L^2\,\widehat{c}^* - 40\,(\widehat{c}^*)^3 \;=\; 24\,L^3 + 2\;L^2\,\widehat{c}^* + 40\,\widehat{c}^*\left[\,L^2 - (\widehat{c}^*)^2\,\right]$$

$$>\; 0 \quad \text{for all } \widehat{c}^* \in [\,0, L\,]. \tag{87}$$

(87) implies that $\eta\,(\widehat{c}^*)$ is a strictly convex function of $\widehat{c}^*$ for all $\widehat{c}^* \in (\,0, L\,)$. Also, from (86) and (87):

$$\eta\,(0) \;=\; 24\,L^5 \;>\; 0\,;\;\; \eta\,(L) \;=\; [\,24 - 40 + 12 + 7 - 2\,]\,L^5 \;=\; L^5 \;>\; 0\,;$$

$$\eta'(0) \;=\; -\,40\,L^4 \;<\; 0\,;\ \text{and}\ \ \eta'(L) \;=\; [\,-\,40 + 24 + 21 - 10\,]\,L^4 \;=\; -\,5\,L^4 \;<\; 0\,. \qquad (88)$$

(87) and (88) imply that $\eta(\widehat{c}^*) > 0$ for all $\widehat{c}^* \in (0, L)$, so (84) holds. Therefore, (84) and (85) imply that $A^* < A^{**}$ if $\overline{p} \in (0,1)$. ■

Explanation. The individuals that gain the most from optional jury service are those with the lowest and highest c realizations. When these realizations are relatively unlikely, there are levels of administrative cost, $A \in (A^*, A^{**})$, for which the population will vote for mandatory jury service even when optional jury service would increase welfare.

Note. This qualitative conclusion might be reversed if the probability that an individual votes is an increasing function of the amount by which the individual's net payoff is affected by the outcome of the vote.

Remark. Some may observe that optional jury service limits the opportunity of individuals with higher c values to be judged by their peers because a relatively large fraction of jurors will have the lower c values. The corresponding countervailing effect is that individuals with lower c values have a better chance of being judged by their peers.

Piecewise Linear, V-Shaped Density

Suppose $g(c)$ is piecewise linear, symmetric around its mean, decreasing for $c \leq E(c)$, and increasing for $c \geq E(c)$. Formally, for $L > 0$ and $g(c) \in [0, 2L]$:

$$g(c) \;=\; \begin{cases} \frac{1}{L} - \frac{c}{L^2} & if \quad 0 \leq c \leq L \\ \\ \frac{c}{L^2} - \frac{1}{L} & if \quad L \leq c \leq 2L. \end{cases} \tag{89}$$

Conclusion 8. *Suppose (89) holds. Then* $A^* \lesseqgtr A^{**}$ *as* $\overline{p} \gtreqless 0.75$.

Proof. When (89) holds, the numerator in the expression for $\widehat{c}^* < L$ in (34) is:

$$\begin{aligned}
& \int_0^{\widehat{c}^*} c\, dG(c) + [1-\overline{p}] \left[\int_{\widehat{c}^*}^{L} c\, dG(c) + \int_L^{2L} c\, dG(c) \right] \\
= \; & \int_0^{\widehat{c}^*} c \left[\frac{1}{L} - \frac{c}{L^2} \right] dc + [1-\overline{p}] \left[\int_{\widehat{c}^*}^{L} c \left(\frac{1}{L} - \frac{c}{L^2} \right) dc + \int_L^{2L} c \left(\frac{c}{L^2} - \frac{1}{L} \right) dc \right] \\
= \; & \int_0^{\widehat{c}^*} \frac{c}{L}\, dc - \int_0^{\widehat{c}^*} \frac{c^2}{L^2}\, dc + [1-\overline{p}] \left[\int_{\widehat{c}^*}^{L} \frac{c}{L}\, dc - \int_{\widehat{c}^*}^{L} \frac{c^2}{L^2}\, dc + \int_L^{2L} \left(\frac{c^2}{L^2} - \frac{c}{L} \right) dc \right] \\
= \; & \left[\frac{c^2}{2L} \right]_0^{\widehat{c}^*} - \left[\frac{c^3}{3L^2} \right]_0^{\widehat{c}^*} + [1-\overline{p}] \left[\left(\frac{c^2}{2L} \right)_{\widehat{c}^*}^{L} - \left(\frac{c^3}{3L^2} \right)_{\widehat{c}^*}^{L} + \left[\frac{c^3}{3L^2} \right]_L^{2L} - \left[\frac{c^2}{2L} \right]_L^{2L} \right] \\
= \; & \frac{(\widehat{c}^*)^2}{2L} - \frac{(\widehat{c}^*)^3}{3L^2} + [1-\overline{p}] \left[\frac{L}{2} - \frac{(\widehat{c}^*)^2}{2L} - \frac{L}{3} + \frac{(\widehat{c}^*)^3}{3L^2} + \frac{8L}{3} - \frac{L}{3} - 2L + \frac{L}{2} \right] \\
= \; & \frac{(\widehat{c}^*)^2}{2L} - \frac{(\widehat{c}^*)^3}{3L^2} + [1-\overline{p}] \left[L - \frac{(\widehat{c}^*)^2}{2L} + \frac{(\widehat{c}^*)^3}{3L^2} \right].
\end{aligned} \tag{90}$$

When (89) holds, the denominator in the expression for $\widehat{c}^*$ in (34) is:

$$\begin{aligned}
G(\widehat{c}^*) + [1-\overline{p}][1 - G(\widehat{c}^*)] \;&=\; \int_0^{\widehat{c}^*} \left[\frac{1}{L} - \frac{c}{L^2} \right] dc + [1-\overline{p}] \left[1 - \int_0^{\widehat{c}^*} \left(\frac{1}{L} - \frac{c}{L^2} \right) dc \right] \\
&=\; \frac{\widehat{c}^*}{L} - \frac{(\widehat{c}^*)^2}{2L^2} + [1-\overline{p}] \left[1 - \frac{\widehat{c}^*}{L} + \frac{(\widehat{c}^*)^2}{2L^2} \right].
\end{aligned} \tag{91}$$

(34), (90), and (91) imply that when (89) holds, the welfare-maximizing value of $\widehat{c}^*$ is determined by:

$$\widehat{c}^* \left[\frac{\widehat{c}^*}{L} - \frac{(\widehat{c}^*)^2}{2\,L^2} + [\,1 - \overline{p}\,] \left(1 - \frac{\widehat{c}^*}{L} + \frac{(\widehat{c}^*)^2}{2\,L^2} \right) \right]$$

$$= \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{3\,L^2} + [\,1 - \overline{p}\,] \left[L - \frac{(\widehat{c}^*)^2}{2\,L} + \frac{(\widehat{c}^*)^3}{3\,L^2} \right]$$

$$\Rightarrow \frac{(\widehat{c}^*)^2}{L} - \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{2\,L^2} + \frac{(\widehat{c}^*)^3}{3\,L^2}$$

$$= [\,1 - \overline{p}\,] \left[L - \frac{(\widehat{c}^*)^2}{2\,L} + \frac{(\widehat{c}^*)^3}{3\,L^2} + \frac{(\widehat{c}^*)^2}{L} - \widehat{c}^* - \frac{(\widehat{c}^*)^3}{2\,L^2} \right]$$

$$\Rightarrow \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - [\,1 - \overline{p}\,] \left[L + \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - \widehat{c}^* \right] = 0$$

$$\Rightarrow \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - L - \frac{(\widehat{c}^*)^2}{2\,L} + \frac{(\widehat{c}^*)^3}{6\,L^2} + \widehat{c}^*$$

$$+ \overline{p} \left[L + \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - \widehat{c}^* \right] = 0$$

$$\Rightarrow -L + \widehat{c}^* + \overline{p} \left[L + \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - \widehat{c}^* \right] = 0$$

$$\Rightarrow \overline{p} \left[L + \frac{(\widehat{c}^*)^2}{2\,L} - \frac{(\widehat{c}^*)^3}{6\,L^2} - \widehat{c}^* \right] = L - \widehat{c}^*$$

$$\Rightarrow \overline{p} \left[6\,(L - \widehat{c}^*)\,L^2 + 3\,L\,(\widehat{c}^*)^2 - (\widehat{c}^*)^3 \right] = 6\,L^2\,[\,L - \widehat{c}^*\,]$$

$$\Rightarrow \overline{p} \left[6\,(L - \widehat{c}^*)\,L^2 + (\widehat{c}^*)^2\,(3\,L - \widehat{c}^*) \right] = 6\,L^2\,[\,L - \widehat{c}^*\,]$$

$$\Rightarrow \overline{p} = \frac{6\,L^2\,[\,L - \widehat{c}^*\,]}{6\,L^2\,[\,L - \widehat{c}^*\,] + (\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]} = \frac{\frac{1}{L^3}\,6\,L^2\,[\,L - \widehat{c}^*\,]}{\frac{1}{L^3}\,6\,L^2\,[\,L - \widehat{c}^*\,] + \frac{1}{L^3}\,(\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]}$$

$$= \frac{6\left[\,1 - \frac{\widehat{c}^*}{L}\,\right]}{6\left[\,1 - \frac{\widehat{c}^*}{L}\,\right] + \left(\frac{\widehat{c}^*}{L}\right)^2 \left[\,3 - \frac{\widehat{c}^*}{L}\,\right]} = \frac{6\,[\,1 - \widetilde{c}\,]}{6\,[\,L - \widetilde{c}\,] + (\widetilde{c})^2\,[\,3 - \widetilde{c}\,]}\,, \qquad (92)$$

where $\widetilde{c} \equiv \frac{\widehat{c}^*}{L}$.

From (50), A^* is defined by:

$$1 - 2\left[G\left(\widehat{c}^* + \frac{A^*}{T}\, a_2\right) - G\left(\widehat{c}^* - \frac{A^*}{T}\, a_1\right)\right] \;=\; 0\,. \tag{93}$$

Observe that:

$$\begin{aligned} A^* \;\gtreqless\; A^{**} \quad &\Leftrightarrow \quad 1 - 2\left[G\left(\widehat{c}^* + \frac{A^{**}}{T}\, a_2\right) - G\left(\widehat{c}^* - \frac{A^{**}}{T}\, a_1\right)\right] \;\gtreqless\; 0 \\ &\Leftrightarrow \quad G\left(\widehat{c}^* + \frac{A^{**}}{T}\, a_2\right) - G\left(\widehat{c}^* - \frac{A^{**}}{T}\, a_1\right) \;\lesseqgtr\; \frac{1}{2}\,. \end{aligned} \tag{94}$$

The first equivalence in (94) holds because the last inequality states that more than half of the population prefers mandatory jury service to optional jury service when $A = A^{**}$. By definition, the same number of individuals prefer mandatory jury service and optional jury service if $A = A^*$. Therefore, A^* must exceed A^{**} and so for $A \in (A^{**}, A^*)$, the majority will favor optional jury service even though welfare would be higher under mandatory jury service.

Because $\frac{A^{**}}{T} = E\{c\} - \widehat{c}^*$ from (52) and $a_2 \equiv \frac{1 - \overline{p}\,[\,1 - G(\widehat{c}^*)\,]}{\overline{p}\, G(\widehat{c}^*)}$ from (49):

$$\begin{aligned} \widehat{c}^* + \frac{A^{**}}{T}\, a_2 \;&=\; \widehat{c}^* + [\, E\{c\} - \widehat{c}^*\,] \left[\frac{1 - \overline{p}\,[\,1 - G(\widehat{c}^*)\,]}{\overline{p}\; G(\widehat{c}^*)} \right] \\ &=\; \widehat{c}^* + [\, E\{c\} - \widehat{c}^*\,] \left[1 + \frac{1 - \overline{p}}{\overline{p}\; G(\widehat{c}^*)} \right] \\ &=\; \widehat{c}^* + E\{c\} - \widehat{c}^* + [\, E\{c\} - \widehat{c}^*\,] \left[\frac{1 - \overline{p}}{\overline{p}\; G(\widehat{c}^*)} \right] \\ &=\; E\{c\} + [\, E\{c\} - \widehat{c}^*\,] \left[\frac{1 - \overline{p}}{\overline{p}\; G(\widehat{c}^*)} \right]. \end{aligned} \tag{95}$$

Because $\frac{A^{**}}{T} = E\{c\} - \widehat{c}^*$ from (52) and $a_1 \equiv \frac{1 - \overline{p}\,[\,1 - G(\widehat{c}^*)\,]}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]}$ from (49):

$$\begin{aligned} \widehat{c}^* - \frac{A^{**}}{T}\, a_1 \;&=\; \widehat{c}^* - [\, E\{c\} - \widehat{c}^*\,] \; \frac{1 - \overline{p}\,[\,1 - G(\widehat{c}^*)\,]}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]} \\ &=\; \widehat{c}^* - [\, E\{c\} - \widehat{c}^*\,] \left[\frac{1}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]} - 1 \right] \\ &=\; \widehat{c}^* + E\{c\} - \widehat{c}^* - [\, E\{c\} - \widehat{c}^*\,] \; \frac{1}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]} \\ &=\; E\{c\} - [\, E\{c\} - \widehat{c}^*\,] \; \frac{1}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]}\,. \end{aligned} \tag{96}$$

(94), (95), and (96) imply:

$$A^* \gtreqless A^{**} \quad \text{as} \quad G\left(E\{c\} + [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_2\right) - G\left(E\{c\} - [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_1\right) \lesseqgtr \frac{1}{2} \tag{97}$$

$$\text{where} \quad \alpha_1 \equiv \frac{1}{\overline{p}\,[\,1 - G(\widehat{c}^{\,*})\,]} \quad \text{and} \quad \alpha_2 \equiv \frac{1 - \overline{p}}{\overline{p}\; G(\widehat{c}^{\,*})}\,. \tag{98}$$

The left hand side of the second inequality in (97) is the area under $g(c)$ for c between $E\{c\} - [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_1$ and $E\{c\} + [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_2$. This area is the sum of the areas under $g(c)$ for c between: (i) $E\{c\} - [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_1$ and L; and (ii) L and $E\{c\} + [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_2$.

From (89), the area under $g(c)$ for c between $E\{c\} - [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_1$ and L is:

$$\begin{aligned}
\int_{L-[\,L-\widehat{c}^{\,*}]\alpha_1}^{L} \left[\frac{1}{L} - \frac{c}{L^2}\right] dc &= \left.\frac{c}{L}\right|_{L-[\,L-\widehat{c}^{\,*}\,]\alpha_1}^{L} - \left.\frac{c^2}{2\,L^2}\right|_{L-[\,L-\widehat{c}^{\,*}]\alpha_1}^{L} \\
&= 1 - \frac{L - [\,L - \widehat{c}^{\,*}\,]\,\alpha_1}{L} - \frac{1}{2} + \frac{\left(L - [\,L - \widehat{c}^{\,*}\,]\,\alpha_1\right)^2}{2\,L^2} \\
&= \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_1}{L} - \frac{1}{2} + \frac{L^2}{2\,L^2} - \frac{2\,L\,[\,L - \widehat{c}^{\,*}\,]\,\alpha_1}{2\,L^2} + \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_1\right)^2}{2\,L^2} \\
&= \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_1}{L} - \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_1}{L} + \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_1\right)^2}{2\,L^2} = \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_1\right)^2}{2\,L^2}\,.
\end{aligned} \tag{99}$$

From (89), the area under $g(c)$ for c between L and $E\{c\} + [\,E\{c\} - \widehat{c}^{\,*}\,]\,\alpha_2$ is:

$$\begin{aligned}
\int_{L}^{L+[\,L-\widehat{c}^{\,*}]\alpha_2} \left[\frac{c}{L^2} - \frac{1}{L}\right] dc &= \left.\frac{c^2}{2\,L^2}\right|_{L}^{L+[\,L-\widehat{c}^{\,*}]\alpha_2} - \left.\frac{c}{L}\right|_{L}^{L+[\,L-\widehat{c}^{\,*}]\alpha_2} \\
&= \frac{\left(L + [\,L - \widehat{c}^{\,*}\,]\,\alpha_2\right)^2}{2\,L^2} - \frac{L^2}{2\,L^2} - \frac{L + [\,L - \widehat{c}^{\,*}]\,\alpha_2}{L} + \frac{L}{L} \\
&= \frac{L^2}{2\,L^2} + \frac{2\,L\,[\,L - \widehat{c}^{\,*}\,]\,\alpha_2}{2\,L^2} + \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_2\right)^2}{2\,L^2} - \frac{1}{2} - 1 - \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_2}{L} + 1 \\
&= \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_2}{L} + \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_2\right)^2}{2\,L^2} - \frac{[\,L - \widehat{c}^{\,*}\,]\,\alpha_2}{L} = \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_2\right)^2}{2\,L^2}\,.
\end{aligned} \tag{100}$$

(97), (99), and (100) imply:

$$\begin{aligned}
A^* \gtreqless A^{**} \quad &\Leftrightarrow \quad \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_1\right)^2}{2\,L^2} + \frac{\left([\,L - \widehat{c}^{\,*}\,]\,\alpha_2\right)^2}{2\,L^2} \lesseqgtr \frac{1}{2} \\
&\Leftrightarrow \quad [\,L - \widehat{c}^{\,*}\,]^2\,(\alpha_1)^2 + [\,L - \widehat{c}^{\,*}\,]^2\,(\alpha_2)^2 \lesseqgtr L^2
\end{aligned}$$

$$\Leftrightarrow \quad [\,L - \widehat{c}^*\,]^2 \left[\,(\alpha_1)^2 + (\alpha_2)^2\,\right] \ \lesseqgtr \ L^2\,. \tag{101}$$

Recall from (91) that $G(\widehat{c}^*) = \frac{\widehat{c}^*}{L} - \frac{(\widehat{c}^*)^2}{2\,L^2}$ when (89) holds. Therefore, From (98):

$$\begin{aligned}
(\alpha_1)^2 + (\alpha_2)^2 &= \left[\frac{1}{\overline{p}\,[\,1 - G(\widehat{c}^*)\,]}\right]^2 + \left[\frac{1-\overline{p}}{\overline{p}\;G(\widehat{c}^*)}\right]^2 \\
&= \frac{1}{(\overline{p})^2}\left\{\left[\frac{1}{[\,1 - G(\widehat{c}^*)\,]^2}\right] + \frac{(1-\overline{p})^2}{[\,G(\widehat{c}^*)\,]^2}\right\} \\
&= \frac{1}{(\overline{p})^2}\left\{\left[\frac{1}{\left[1 - \frac{\widehat{c}^*}{L} + \frac{(\widehat{c}^*)^2}{2\,L^2}\right]^2}\right] + \left[\frac{(1-\overline{p})^2}{\left[\frac{\widehat{c}^*}{L} - \frac{(\widehat{c}^*)^2}{2\,L^2}\right]^2}\right]\right\} \\
&= \frac{1}{(\overline{p})^2}\left\{\left[\frac{1}{\left[\frac{2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2}{2\,L^2}\right]^2}\right] + \left[\frac{(1-\overline{p})^2}{\left[\frac{2\,L\,\widehat{c}^* - (\widehat{c}^*)^2}{2\,L^2}\right]^2}\right]\right\} \\
&= \frac{4\,L^4}{(\overline{p})^2\left[\,2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2\,\right]^2} + \frac{4\,L^4\,[\,1-\overline{p}\,]^2}{(\overline{p})^2\left[\,2\,L\,\widehat{c}^* - (\widehat{c}^*)^2\,\right]^2}\,.
\end{aligned} \tag{102}$$

From (92):

$$\frac{1-\overline{p}}{\overline{p}} = \frac{1 - \frac{6\,L^2[\,L-\widehat{c}^*\,]}{6\,L^2[\,L-\widehat{c}^*\,]+(\widehat{c}^*)^2[\,3\,L-\widehat{c}^*\,]}}{\frac{6\,L^2[\,L-\widehat{c}^*\,]}{6\,L^2[\,L-\widehat{c}^*\,]+(\widehat{c}^*)^2[\,3\,L-\widehat{c}^*\,]}} = \frac{(\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]}{6\,L^2\,[\,L - \widehat{c}^*\,]}\,. \tag{103}$$

(92), (102), and (103) imply:

$$\begin{aligned}
(\alpha_1)^2 + (\alpha_2)^2 &= \frac{4\,L^4}{(\overline{p})^2\left[\,2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2\,\right]^2} + \left[\frac{(\widehat{c}^*)^2\,(3\,L - \widehat{c}^*)}{6\,L^2\,(L - \widehat{c}^*)}\right]^2 \frac{4\,L^4}{\left[\,2\,L\,\widehat{c}^* - (\widehat{c}^*)^2\,\right]^2} \\
&= \frac{4\,L^4}{(\overline{p})^2\left[\,2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2\,\right]^2} + \frac{(\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]^2}{9\,[\,L - \widehat{c}^*\,]^2\,[\,2\,L - \widehat{c}^*\,]^2} \\
&= \frac{\left[\,6\,(L - \widehat{c}^*)\,L^2 + (\widehat{c}^*)^2\,(3\,L - \widehat{c}^*)\,\right]^2}{36\,L^4\,[\,L - \widehat{c}^*\,]^2}\,\frac{4\,L^4}{\left[\,2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2\,\right]^2} + \frac{(\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]^2}{9\,[\,L - \widehat{c}^*\,]^2\,[\,2\,L - \widehat{c}^*\,]^2} \\
&= \frac{\left[\,6\,(L - \widehat{c}^*)\,L^2 + (\widehat{c}^*)^2\,(3\,L - \widehat{c}^*)\,\right]^2}{9\,[\,L - \widehat{c}^*\,]^2\left[\,2\,L^2 - 2\,L\,\widehat{c}^* + (\widehat{c}^*)^2\,\right]^2} + \frac{(\widehat{c}^*)^2\,[\,3\,L - \widehat{c}^*\,]^2}{9\,[\,L - \widehat{c}^*\,]^2\,[\,2\,L - \widehat{c}^*\,]^2}
\end{aligned}$$

$$\Rightarrow \quad [\,L - \widehat{c}^*\,]^2 \left[\,(\alpha_1)^2 + (\alpha_2)^2\,\right]$$

$$= \frac{\left[6 (L - \widehat{c}^*) L^2 + (\widehat{c}^*)^2 (3 L - \widehat{c}^*) \right]^2}{9 \left[2 L^2 - 2 L \widehat{c}^* + (\widehat{c}^*)^2 \right]^2} + \frac{(\widehat{c}^*)^2 [3 L - \widehat{c}^*]^2}{9 [2 L - \widehat{c}^*]^2} . \tag{104}$$

(101) and (104) imply:

$$[L - \widehat{c}^*]^2 \left[(\alpha_1)^2 + (\alpha_2)^2 \right] \lesseqgtr L^2 \Leftrightarrow \widetilde{\varphi}(\widehat{c}^*) \gtreqless 0, \text{ where, for } \widehat{c}^* \in [0, L], \tag{105}$$

$$\widetilde{\varphi}(\widehat{c}^*) \equiv L^2 - \frac{\left[6 (L - \widehat{c}^*) L^2 + (\widehat{c}^*)^2 (3 L - \widehat{c}^*) \right]^2}{9 \left[2 L^2 - 2 L \widehat{c}^* + (\widehat{c}^*)^2 \right]^2} - \frac{(\widehat{c}^*)^2 [3 L - \widehat{c}^*]^2}{9 [2 L - \widehat{c}^*]^2}$$

$$= L^2 \left\{ 1 - \frac{\left[6 \left(1 - \frac{\widehat{c}^*}{L}\right) L^2 + (\widehat{c}^*)^2 \left(3 - \frac{\widehat{c}^*}{L}\right) \right]^2}{9 \left[2 L^2 - 2 L \widehat{c}^* + (\widehat{c}^*)^2 \right]^2} - \frac{(\widehat{c}^*)^2 \left[3 - \frac{\widehat{c}^*}{L} \right]^2}{9 [2 L - \widehat{c}^*]^2} \right\}$$

$$= L^2 \left\{ 1 - \frac{\frac{1}{L^4} \left[6 \left(1 - \frac{\widehat{c}^*}{L}\right) L^2 + (\widehat{c}^*)^2 \left(3 - \frac{\widehat{c}^*}{L}\right) \right]^2}{\frac{1}{L^4} 9 \left[2 L^2 - 2 L \widehat{c}^* + (\widehat{c}^*)^2 \right]^2} - \frac{\frac{1}{L^2} (\widehat{c}^*)^2 \left[3 - \frac{\widehat{c}^*}{L} \right]^2}{\frac{1}{L^2} 9 [2 L - \widehat{c}^*]^2} \right\} = L^2 \varphi(\widetilde{c}) \tag{106}$$

$$\text{where } \varphi(\widetilde{c}) \equiv 1 - \frac{\left[6 (1 - \widetilde{c}) + (\widetilde{c})^2 (3 - \widetilde{c}) \right]^2}{9 \left[2 - 2 \widetilde{c} + (\widetilde{c})^2 \right]^2} - \frac{(\widetilde{c})^2 [3 - \widetilde{c}]^2}{9 [2 - \widetilde{c}]^2} \text{ and } \widetilde{c} = \frac{\widehat{c}^*}{L} . \tag{107}$$

(101) and (106) imply that $A^* \lesseqgtr A^{**}$ as $\varphi(\widetilde{c}) \lesseqgtr 0$.

(107) implies:

$$\varphi(0) = 1 - \frac{[6]^2}{9 [2]^2} = 0 \quad \text{and} \quad \varphi(1) = 1 - \frac{[2]^2}{9} - \frac{[2]^2}{9} = \frac{1}{9} > 0 . \tag{108}$$

Furthermore, it can be verified that for $\widetilde{c} \in (0, 1]$, $\varphi(\widetilde{c}) \lesseqgtr 0$ as $\widetilde{c} \lesseqgtr \widetilde{c}_1 \approx 0.585786$. In addition, (92) implies that $\overline{p} = 0.75$ when $\widetilde{c} = \widetilde{c}_1$. Also, from (92):

$$\frac{\partial \overline{p}}{\partial \widehat{c}^*} \stackrel{s}{=} -6 L^2 [L - \widehat{c}^*] - (\widehat{c}^*)^2 [3 L - \widehat{c}^*] - [L - \widehat{c}^*] \left[-6 L^2 + 6 L \widehat{c}^* - 3 (\widehat{c}^*)^2 \right]$$

$$= - (\widehat{c}^*)^2 [3 L - \widehat{c}^*] - [L - \widehat{c}^*] \left[6 L \widehat{c}^* - 3 (\widehat{c}^*)^2 \right]$$

$$= - 3 L (\widehat{c}^*)^2 + (\widehat{c}^*)^3 - 6 L^2 \widehat{c}^* + 3 L (\widehat{c}^*)^2 + 6 L (\widehat{c}^*)^2 - 3 (\widehat{c}^*)^3$$

$$= - 6 L \widehat{c}^* [L - \widehat{c}^*] - 2 (\widehat{c}^*)^3 < 0 .$$

Because $\overline{p}$ and $\widehat{c}^*$ vary inversely, it follows that $\varphi(\widetilde{c}) \lesseqgtr 0$ as $p \gtreqless 0.75$. ■

Conclusion 9. *Suppose* $g(\underline{c}) > 0$. *Then as* $\overline{p} \to 1$, $A^* \gtreqless A^{**} \Leftrightarrow E(c) \lesseqgtr c^M$.

Proof. Recall from (94) that:

$$A^* \gtreqless A^{**} \quad \Leftrightarrow \quad G(\widehat{c}^* + [E(c) - \widehat{c}^*]\, a_2) - G(\widehat{c}^* - [E(c) - \widehat{c}^*]\, a_1) \;\lesseqgtr\; \frac{1}{2} \tag{109}$$

$$\text{where} \quad a_1 \equiv \frac{1 - \overline{p}\,[1 - G(\widehat{c}^*)]}{\overline{p}\,[1 - G(\widehat{c}^*)]} \quad \text{and} \quad a_2 \equiv \frac{1 - \overline{p}\,[1 - G(\widehat{c}^*)]}{\overline{p}\, G(\widehat{c}^*)}. \tag{110}$$

Observe that:

$$a_1 \equiv \frac{1 - \overline{p}\,[1 - G(\widehat{c}^*)]}{\overline{p}\,[1 - G(\widehat{c}^*)]} \to \frac{G(\widehat{c}^*)}{1 - G(\widehat{c}^*)} \to 0 \;\text{ as } \overline{p} \to 1. \tag{111}$$

This conclusion in (111) holds because $\widehat{c}^* \to \underline{c}$ as $\overline{p} \to 1$, from Conclusion 4.

We will now prove that:

$$a_2 \equiv \frac{1 - \overline{p}\,[1 - G(\widehat{c}^*)]}{\overline{p}\, G(\widehat{c}^*)} = 1 + \frac{1 - \overline{p}}{\overline{p}\, G(\widehat{c}^*)} \to 1 \;\text{ as } \overline{p} \to 1. \tag{112}$$

From L'Hopital's rule:

$$\begin{aligned} \lim_{\overline{p} \to 1} \left[\frac{1 - \overline{p}}{\overline{p}\, G(\widehat{c}^*)} \right] &= \lim_{\overline{p} \to 1} \left[\frac{\frac{\partial}{\partial \overline{p}}(1 - \overline{p})}{\frac{\partial}{\partial \overline{p}}(\overline{p}\, G(\widehat{c}^*))} \right] = \lim_{\overline{p} \to 1} \left[\frac{-1}{G(\widehat{c}^*) + \overline{p}\, \frac{\partial}{\partial \overline{p}} G(\widehat{c}^*)} \right] \\ &= \lim_{\overline{p} \to 1} \left[\frac{-1}{G(\widehat{c}^*) + \overline{p}\, g(\widehat{c}^*)\, \frac{\partial \widehat{c}^*}{\partial \overline{p}}} \right]. \end{aligned} \tag{113}$$

Recall from (48) that:

$$\frac{\partial \widehat{c}^*}{\partial \overline{p}} = \frac{\widehat{c}^*\,[1 - G(\widehat{c}^*)] - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c)}{G(\widehat{c}^*) + [1 - \overline{p}]\,[1 - G(\widehat{c}^*)]}. \tag{114}$$

Because $\widehat{c}^* \to \underline{c}$ as $\overline{p} \to 1$ (from Conclusion 4), it follows that as $\overline{p} \to 1$:

$$\widehat{c}^*\,[1 - G(\widehat{c}^*)] - \int_{\widehat{c}^*}^{\overline{c}} c\, dG(c) \to \underline{c} - \int_{\underline{c}}^{\overline{c}} c\, dG(c) < 0, \text{ and} \tag{115}$$

$$G(\widehat{c}^*) + [1 - \overline{p}]\,[1 - G(\widehat{c}^*)] \to 1 - \overline{p} \to 0. \tag{116}$$

Because $G(\widehat{c}^*) + [1 - \overline{p}]\,[1 - G(\widehat{c}^*)] \geq 0$, (114), (115), and (116) imply:

$$\frac{\partial \widehat{c}^*}{\partial \overline{p}} \to -\infty \;\text{ as } \overline{p} \to 1. \tag{117}$$

(113) and (117) imply that if $g(\underline{c}) > 0$, then:

$$\lim_{\overline{p}\to 1}\left[\frac{1-\overline{p}}{\overline{p}\,G\left(\widehat{c}^{*}\right)}\right] \;=\; \lim_{\overline{p}\to 1}\left[\frac{-1}{G\left(\widehat{c}^{*}\right)+\overline{p}\;g\left(\widehat{c}^{*}\right)\,\frac{\partial\widehat{c}^{*}}{\partial\overline{p}}}\right] \;=\; 0\,,$$

so the conclusion in (112) holds.

(109), (111), and (112) imply that as $\overline{p}\to 1$:

$$A^{*} \gtreqless A^{**} \;\Leftrightarrow\; G\left(\underline{c}+\left[E\left(c\right)-\underline{c}\right]\right)-G\left(\underline{c}\right) \lesseqgtr \frac{1}{2} \;\Leftrightarrow\; G\left(E\left(c\right)\right) \lesseqgtr \frac{1}{2}\,. \tag{118}$$

Let c^{M} be the median of the density $g\left(c\right)$. Then (118) implies that as $\overline{p}\to 1$:

$$A^{*} \gtreqless A^{**} \;\Leftrightarrow\; G\left(E\left(c\right)\right) \lesseqgtr G\left(c^{M}\right) \;\Leftrightarrow\; E\left(c\right) \lesseqgtr c^{M}\,. \quad \blacksquare \tag{119}$$

Corollary. Suppose $g(\underline{c}) > 0$ and $E\left(c\right) > c^{M}$. Then as $\overline{p}\to 1\,,\; A^{*} < A^{**}$. Under these circumstances, if $A \in (A^{*}, A^{**})$, then a majority of potential jurors oppose Optional Jury Service even though it generates a higher level of welfare than Mandatory Jury service.

Interpretation and Explanation of Conclusion 9

Observe that as $\overline{p}\to 1$, $G\left(\widehat{c}^{*}-\left[E\left(c\right)-\widehat{c}^{*}\right]a_{1}\right)\to G\left(\widehat{c}^{*}\right)\to 0$ and $G\left(\widehat{c}^{*}+\left[E\left(c\right)-\widehat{c}^{*}\right]a_{2}\right)$ $\to G\left(E\left(c\right)\right)$. Consequently, as $\overline{p}\to 1$, all individuals with $c < E\left(c\right)$ prefer mandatory jury service (MJS) to an optimally-designed optional jury service (OJS) program (and the individual with $c = \underline{c}$ is indifferent between the two programs), whereas individuals with $c > E\left(c\right)$ prefer the OJS program to MJS. Under a majority rule policy, the choice between OJS and MJS is determined by the median voter. Consequently, if $c^{M} < E(c)$, voters may reject OJS even when it would generate a higher level of welfare than MJS.

To understand why all individuals with $c < E\left(c\right)$ prefer MJS to an optimally-designed OJS program and the individual with $c = \underline{c}$ is indifferent between the two programs as $\overline{p}\to 1$, suppose the $\underline{c}$ type strictly preferred the OJS program to MJS. Then the payment to perform jury service (w) could be reduced to eliminate the strict preference of the $\underline{c}$ type. The reduction in w would harm the $\underline{c}$ type but would benefit the $\widehat{c}$ type by a greater amount. The increased benefit to the $\widehat{c}$ type would arise because he would pay less to opt out of jury service and he would avoid a relatively high cost ($\widehat{c} > \underline{c}$) by doing so. Thus, the reduction in w would increase expected welfare.

A different calculation may be relevant when $\overline{p} < 1$. Now a reduction in w would definitely harm the $\underline{c}$ type (and all others who chose to perform jury service under the OJS program), but would only benefit the $\widehat{c}$ type (and higher c types) probabilistically. With probability $1-\overline{p}$, no benefit from reducing w would arise because the request to opt out of jury service would be denied, and the individual that made the request would receive only the reduced payment w to perform jury service. In essence, it appears that when $\overline{p} < 1$, there is a deadweight loss that can render it sub-optimal to reduce w to the point where the $\underline{c}$ type is indifferent between OJS and MJS.

Observations

1. Numerical solutions for an inverted V shaped density suggest that $A^* < A^{**}$ when $\overline{p}$ is sufficiently close to 1. It would be useful to know if this conclusion holds whenever $E(c) = c^M$.

2. Numerical solutions suggest that (112) holds if $g(\underline{c}) = 0$. It would be useful to know if this result holds in general.

3. Conclusion 9 considers the case where $\overline{p} \to 1$ rather than $\overline{p} = 1$ because, from Assumption (2), we need $N > \frac{T}{1-\overline{p}}$. We might interpret the limiting case as one in which an individual's request to opt out of jury service is honored, except in exceptional circumstances.

4. We need to consider whether, in practice, the mean c is likely to exceed, equal, or be less than the median c.

   Note that a few individuals may have extremely high costs of performing jury service, which could make $E(c)$ exceed c^M.