Relational Contracting, Negotiation, and External Enforcement

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September 2017 version for Repeated Games Conference at Konstanz
Preliminary/incomplete – please do not circulate beyond the conference.

Abstract

We study relational contracting and renegotiation in environments with external enforcement of long-term contractual arrangements. The external part of a long-term contract governs the stage games the contracting parties will play in the future (depending on verifiable stage-game outcomes) until they renegotiate. In a *contractual equilibrium*, the parties choose their individual actions rationally, they jointly optimize when selecting a contract, and they take advantage of their relative bargaining power. Our main result is that in a wide variety of settings, in each period of a contractual equilibrium the parties negotiate to a single *semi-stationary* externally enforced arrangement, meaning the same specifications for every future period but special terms for the current period. Over time, essentially the parties adjust only their contractual specifications for the current period. For examples, in a simple principal-agent model with a choice of costly monitoring technology, the optimal contract specifies mild monitoring for the current period but intense monitoring for future periods. Because the parties renegotiate in each new period, intense monitoring arises only off the equilibrium path after a failed renegotiation.

While long-term economic relationships are often governed in part by legal contract terms, contracting parties also agree to provide incentives through their own endogenous behavior, generating rewards and punishments in their continuation play. Macaulay (1963) observed that contractual relationships between US firms were often loosely specified in legal terms, but that firms persisted using such loosely specified contracts, seemingly anticipating that in ongoing business relationships they would be able to work something out, should disagreements arise (Malcomson 2013). The literature on relational contracting has developed this theme, with applications to a large variety of economic environments.

This paper is the first to explicitly account for both recurring negotiations and long-term externally enforced contracts in such ongoing relationships. We identify key features of optimal contractual relationships when parties can write long-term legal contracts that are constrained to be incomplete and renegotiable. In addition to providing a general framework and theoretical results, we explain some actual practices, such as the interplay of long-run and short-run contractual provisions, stationary terms, and the allocation of control rights.

We view the *contract* between parties as comprising both an *external part*, which is enforced by the courts or other external referee; and an *internal part*, which is self-enforced by the parties' continuation play.[1] We model an environment in which both parts of the contract are renegotiable. Though the environment is stationary, the ability to write arbitrary external contract terms introduces the possibility of endogenous non-stationarity: in the current period, the external terms agreed upon in the prior period can be changed only by mutual agreement, and thus constitute a payoff-relevant state variable. What should the contracting parties do in such an environment? Should they write a stationary contract, or a non-stationary contract?

The prior literature establishes that without external enforcement if the parties can pay monetary transfers that enter their payoffs linearly, then optimal behavior on the equilibrium path is stationary (see, e.g., Levin 2003; Miller and Watson 2013).[2] Introducing external enforcement to an otherwise stationary environment, we find that while it is optimal for the contracting parties to write the same contract every time they renegotiate, the external part of that contract is itself non-stationary. If the external enforcer can compel monetary transfers as a function of verifiable outcomes (or if no outcomes are verifiable), then the

[1] In the literature, the external/internal dichotomy is variously called "explicit/implicit," "formal/informal," or "legal/relational." The terminology we prefer focuses attention on the source of the enforcement power. While the "legal/relational" terminology does so as well, we prefer to think of a "relational contract" as encompassing both the internal and external parts of the contract.

[2] In the absence of external enforcement, a relational contract is either a perfect public equilibrium (e.g., Levin 2003) or a contractual equilibrium (Miller and Watson 2013) of an infinitely repeated game.

non-stationarity takes a special form. Optimally, the long-term external part, which governs future periods, should be stationary; but the short-term external part, which governs the current period, should be special. We call such a contract *semi-stationary*. Intuitively, the parties choose the long-term external part to maximize the power of incentives, while they choose the the short-term external part to maximize their joint payoffs given the power of incentives available to them. Since they anticipate renegotiating to the same contract in each new period, along the equilibrium path they always operate under the short term external part of the contract.

These features are illustrated in several settings. We start with a simple principal-agent model with a choice of a costly and externally enforceable monitoring technology. The optimal semi-stationary contract specifies mild monitoring for the current period, but intense monitoring for future periods. Since the parties renegotiate in each new period, intense monitoring is enforced only out of equilibrium after a failed renegotiation. The intense monitoring affects disagreement payoffs in such a way that the span of available continuation payoffs, accounting for renegotiation, is enlarged. The larger span enables the parties to save on costly monitoring in the current period.

Our modeling approach allows for a broad range of external enforcement capabilities. The external enforcer can impose a stage game for the contracting parties to play, and which stage game is imposed can depend on the verifiable outcomes in prior periods. Thus the enforcer's capabilities are defined by the set of stage games it has available to impose, where each stage game includes a partition defining the extent to which the enforcer can verify outcomes.

Our solution concept is *contractual equilibrium* (Miller and Watson 2013), applied to a hybrid repeated game in which each period contains two phases: a cooperative *negotiation phase* and a non-cooperative *action phase*. In the negotiation phase, the players reach an agreement that satisfies the generalized Nash (1950) bargaining solution, where the bargaining set contains all valid continuation payoff vectors and the disagreement point is determined in equilibrium. In the action phase, the players' actions should depend only on the public history and must satisfy individual incentive constraints, just as in a perfect public equilibrium. Since Miller and Watson (2013) provide fully non-cooperative foundations using cheap-talk bargaining and axiomatic equilibrium selection, in this paper we restrict attention to the hybrid cooperative/non-cooperative game. (Generalizing Miller and Watson's fully non-cooperative framework to allow for external enforcement would be notationally cumbersome, but conceptually straightforward.)

In addition to the simple principal-agent setting with monitoring, we apply our framework to partnerships and multitasking. A common theme is that the semi-stationary nature of the equilibrium contract implies that strict terms in the externally enforced part of the contract, are routinely renegotiated to other (and often milder) terms. In equilibrium, the strict legal terms are thus actually never enforced. It is noteworthy that this type of behavior is often observed in reality. For instance, it is common practice in many organizations to have strict formal rules for employees, e.g. with respect to attendance and procedures at work, but to allow and accept considerable flexibility regarding adherence to these rules. Our framework provides an explanation for such practices. It should be noted that Iossa and Spagnolo (2011) also provide an explanation based on the interplay of relational and legal contracts; the differences between their approach and ours are discussed below.

It is well known (cf. Bernheim and Whinston 1998) that strategic flexibility can be valuable when some, but not all actions for the players can be externally enforced. In our applications we show that such valuable flexibility can be achieved by letting the externally enforced terms of the contract take the form of options. In the monitoring example we show that allowing the principal to select between two externally enforced options for monitoring (very strict and very mild, respectively) improves equilibrium welfare relative to specifying a contractually fixed level of monitoring. We also show that it matters whether it is the principal or the agent who has the right to select between options; thus decision rights are shown to matter in such settings. [3]

Our applications also show that, while the long term externally enforced contract is in general modified in renegotiations each period, this need not be the case in all environments. In a multitask setting, where an agent supplies efforts on two tasks with, respectively verifiable and non-verifiable but observable outputs, (e.g. quantity and quality, respectively), we show that under some conditions the optimal contract has the following features. The externally enforced part of the contract takes the form of a payment schedule, conditional on the verifiable output (quantity). Incentives for the other task (promoting quality) are provided internally, and are thus self-enforced. The externally enforced payment schedule

[3] Baker, Gibbons, and Murphy (2011) also demonstrate how allocation of control rights matters in relational contracting, but via a channel much different from ours. They analyze how governance structures (allocations of control) can facilitate relational contracts that improve on spot transactions in settings where such transactions would produce inefficient adaptation to changing circumstances. They show, among other things, that the optimal governance structure for implementing a given relational contract minimizes the maximum aggregate reneging temptation created by that relational contract. Relatedly, Barron, Gibbons, Gil, and Murphy (2015) analyze self-enforcing agreements that facilitate efficient adaptation (called relational adaptation), and show how these agreements combined with formal contracting, induce state-dependent decision-making that improves upon the expected payoffs under either formal contracting or relational contracting alone. Their theoretical model assumes stationarity of equilibrium strategies and Nash reversion, i.e., reversion after a deviation to the equilibrium of the one-period game for all the future.

remains fixed and is never renegotiated. The parties agree to the same quantity and associated payment each period, but realize that other quantities and payments from this schedule will be implemented should they fail to reach agreement.

Related literature

In the words of Malcomson (2013): ”The literature on relational contracts is concerned with the impact of the on-going nature of the relationship on trade between the parties, on their payoffs, on the nature of any legally enforceable contract that is used to supplement the relational contract, and on the design of organizations.” This paper focuses on the nature of the externally enforced part of the contract, and on its implications for the overall relationship. Several previous papers, including Baker, Gibbons, and Murphy (1994, 2002), Schmidt and Schnitzer (1995), Kvaløy and Olsen (2009), and Iossa and Spagnolo (2011), have dealt with this issue. In contrast to these previous analyses, we allow the players to negotiate (and renegotiate) over how to play and how to select the terms of the external enforcement. Technically this is achieved by extending the concept of contractual equilibrium for infinite horizon games in Miller and Watson (2013) to environments where some elements of the game can be externally enforced. This approach addresses the question of how agents initiate and manage their relationship, including how their agreements evolve after deviations and disagreements.

Most prior analyses of interactions between self enforced and externally enforced contractual terms, including those cited above, assumed that the parties are limited to external enforcement following any deviation. This assumption often implies that improvements in external enforcement can reduce equilibrium welfare by constraining the severity of punishments. In contractual equilibrium, however, the parties can reevaluate their entire relationship when they bargain, and this implies that equilibrium welfare always should never decrease with improved external enforcement. This prediction is in line with recent empirical studies that find complementarity between externally enforced and self-enforced contracts (Beuve and Saussier (2012); Lazzarini, Miller, and Zenger (2004); Ryall and Sampson (2009); Poppo and Zenger (2002)).

As noted above, some of our results have a similar flavor to those of Iossa and Spagnolo (2011). They point out that it is common practice for contracting parties to write legally enforceable contracts that contain inefficient clauses, but where these clauses are ignored in equilibrium. They explain practice based on the observation that an inefficient legally enforceable contract can be used as a credible threat to sustain a more efficient relational contract.[4] Given that the legal contract can be written such that it destroys value if enforced, the analysis verifies that it enhances informal relational (and even static) contracting.

Apart from our model being considerably more general in many respects, a major difference between our approach and theirs is the way the parties are assumed to bargain and renegotiate. Iossa-Spagnolo allows for renegotiation after a deviation, but where disagreement in that bargaining implies adherence to the inefficient formal contract—which is assumed to be stationary—in all future periods. Our framework instead insists on internal bargaining consistency, implying that the outcome after a disagreement must be an equilibrium in the game where the parties may bargain in all future periods. In this setting we show that the externally enforced terms of the overall contract are indeed stationary, and that these terms are generally renegotiated (and thus ”ignored”) in each period before actions are taken.

The analysis of relational contracts was initiated by Klein and Leffler (1981), Shapiro and Stiglitz (1984), Bull (1987) and MacLeod and Malcomson (1989).[5] Levin (2003) showed that with transfers, optimal contracts in a time invariant environment can be taken to be stationary. The key reason for stationarity is that the combination of quasi-linear utility and monetary transfers allow discretionary payments to substitute for variations in continuation payoffs. Levin also observed that optimal stationary contracts are ”strongly optimal” in the sense that, for any feasible history, the continuation contract onwards is optimal. This is a variant of renegotiation proofness in relational contracting, a theme further pursued by Goldlücke and Kranz (2012). They assume perfect monitoring (and no external enforcement), and show that Pareto-optimal subgame perfect payoffs and renegotiation proof payoffs can generally be found by restricting attention to a simple class of stationary contracts.

Relative to renegotiation proofness, contractual equilibrium entails a different approach to equilibrium selection. The contrasts are discussed in depth in Miller and Watson (2013). Suffice it here to point out that, unlike contractual equilibrium, renegotiation proofness rules out renegotiation rather than modeling it explicitly, and thus does not account for the possibility of disagreement.[6] By incorporating bargaining power in a tractable way, contractual equilibrium yields different testable implications about contractual outcomes

[4]This is formally verified in a multi-task principal-agent model, first in a basic setting where the parties are committed to the legal contract for the long term, and then extended to settings where the parties can (to some extent) renegotiate.

[5]While the formal literature starts with Klein and Leffler, the concept of relational contracts had was first defined and explored by legal scholars (e.g., Macaulay 1963; Macneil 1978).

[6]Safronov and Strulovici (2016) also model renegotiation explicitly and allow for disagreements in a repeated game setting, but do not consider external enforcement.

than does renegotiation proofness.

Optimal relational contracts in time invariant environments may be non-stationary due to, e.g., limited liability (Fong and Li 2015) or persistent private information and limited enforcement (Martimort, Semenov, and Stole 2016). No such features are present in the model analyzed here, but we show that limited external enforcement alone may make the equilibrium contract non-stationary. (With no external enforcement, the contractual equilibrium is fully stationary, as shown in Miller and Watson 2013.) The semi-stationary contract we find specifies the same externally enforced terms for every future period, but special terms for the current period.

It is worth noting that semi-stationarity distinguishes our equilibrium contract from the fully stationary externally enforced contracts often assumed in relational contracting. Che and Yoo (2001), for instance, allow only stationary externally enforced wage contracts in their analysis of team incentives. [7]

Bernheim and Whinston (1998) analyze dynamic contracting problems with incomplete contracts and renegotiation of both externally and internally enforced terms, but confine attention to finite-horizon (two period) games. Their main point is to show that, when some aspects of performance is unverifiable, it is often optimal to leave other verifiable aspects of performance unverifiable; thus leaving optimal contracts less complete than they could have been. The reason is that the incompleteness gives flexibility to create rewards and punishments for actions in previous periods; actions that are by assumption non-verifiable, and hence must be self-enforced. Such strategic ambiguity may also appear in our contractual equilibrium; and in some applications in the form of options, where the contract allows a party to select from a set of verifiable alternatives rather than specifying exactly which verifiable alternative the party should select.

A considerable literature has investigated the implications of renegotiations in incomplete contract settings. Classic papers such as Hart and Moore (1988), Hart and Moore (1999), Aghion, Dewatripont, and Rey (1994), Noldeke and Schmidt (1995), Che and Hausch (1999), Segal (1999), Maskin and Tirole (1999) are surveyed in Bolton and Dewatripont (2005). Much of this literature has focused on the hold-up problem, where the parties make relationship-specific investments and may subsequently renegotiate the division of the resulting surplus, which can lead to underinvestment if not all parties realize the full marginal benefit of their investments. Given complete information ex post, it has been shown that the problem can in some cases be solved by contractual design of the renego-

[7] Their work also differs in other ways—they assume limited liability and no monetary transfers among the agents—so we cannot make a direct comparison.

tiation process (Aghion, Dewatripont, and Rey (1994), Noldeke and Schmidt (1995)). In other cases, such as when there are bilateral direct externalities (Che and Hausch (1999)) or the environment is ”complex” (Segal (1999), Hart and Moore (1999)), then contracts may achieve nothing when renegotiation cannot be precluded. In those cases the optimal contract is the ”null contract”, and the hold up problem may then be severe.[8] Along the same lines, and building on Maskin and Moore (1999), Segal and Whinston (2002) characterize implementable investments and optimal contracts (mechanisms) when the parties have complete information ex post and renegotiate to efficient outcomes following the play of the mechanism. Evans (2008), on the other hand, shows that if production and trade can be delayed while negotiation is taking place (in which case renegotiation will not yield ex post efficiency), then there are contracts that will induce efficient investments ex ante even when there are bilateral externalities or the environment is complex.

In contrast to most of this (mechanism design) literature, we take the negotiation protocol as exogenously given[9] (see Miller and Watson 2013 for the non-cooperative specification), assume that negotiations take place only before actions in each period, and consider an infinite repetition of this structure. Following the relational contracting approach, we then analyze what internal and external enforcement can achieve in this setting.

1 Example: Choice of a Monitoring Technology

As illustration, consider the relationship between a worker and a manager, where the extent to which the manager can monitor worker's effort is determined by a costly monitoring technology that can be externally enforced—for instance by a third party who is hired to observe the worker.

The worker (player 1) and the manager (player 2) interact over discrete time periods with an infinite horizon and a shared discount factor δ. Each period comprises two phases:

- the *negotiation phase*, where the players can establish or revise their contract, as well as make immediate monetary transfers; and
- the *action phase*, where productive interaction occurs.

A contract has two components, an externally enforced part and a self-enforced part, the latter which specifies how the players coordinate their future behavior. External enforce-

[8] But even in such settings, Maskin and Tirole (1999) show that if the parties are risk averse, one can exploit this to construct mechanisms that achieve first best implementation.

[9] In Evans (2008) the negotiation protocol is also exogenous, while negotiations take place ex post actions, the latter being selected only once.

ment is discussed below. Let the immediate monetary transfer in the negotiation phase be denoted $m_1 \in \mathbb{R}$, indicating the amount the manager pays the worker.

In the action phase, the worker chooses her action a: either low effort ($a = 0$), or high effort ($a = 1$). High effort imposes a personal cost of $\beta \in (0, 1)$ on the worker and yields a benefit of 1 to the manager, both in monetary terms. The players jointly observe a signal x generated by a monitoring technology with accuracy parameter $\mu \in [0, 1]$. If the worker exerts high effort then the signal realization is $x = 1$ for sure, but if the worker exerts low effort then the signal realization is either $x = 1$, with probability $1 - \mu$, or $x = 0$, with probability μ. The monitoring technology imposes a cost of $k(\mu)$ on the manager that is strictly increasing in μ and satisfies $\beta + k(1) \leq 1$ (so high effort with maximal monitoring generates higher welfare than low effort with minimal monitoring).

At the end of a period, the players publicly observe the signal generated by the monitoring technology. However, only the worker observes her own effort choice a. To keep things simple we also assume that the manager does not observe his own stage game payoff.[10] We assume that the players can take advantage of arbitrary public randomization devices to coordinate their play.

1.1 Perfect public equilibrium with fixed monitoring technology

Let us begin the analysis by holding aside contracting to consider perfect public equilibria of the game in which the players can make voluntary transfers (but otherwise do not communicate) in the negotiation phase. The worker is willing to exert high effort each period if doing so leads to future transfers from the manager, and if a deviation would be detected and punished with high enough probability and severity. If such cooperation can be achieved then it can be supported using a grim-trigger strategy profile in which the worker chooses $a = 1$ and the manager pays m_1 in each period, so long as the signal realization was $x = 1$ and the manager paid m_1 in all previous periods; otherwise there is no transfer and the worker chooses $a = 0$.

There are two conditions for a high-effort equilibrium. First, the worker must not have the incentive to deviate once to $a = 0$ along the equilibrium path. Her incentive constraint for this deviation is

$$(1 - \delta)(m_1 + 0) + \delta\big(\mu \cdot 0 + (1 - \mu)(m_1 - \beta)\big) \leq m_1 - \beta. \tag{1}$$

[10] Alternatively, one could assume that the manager's payoff depends only on the monitoring signal, equaling 1 if $x = 1$ and $-\frac{\mu}{1-\mu}$ if $x = 0$.

Similarly, the manager must not profit from deviating once to $m_1 = 0$. Since the monitoring cost is unavoidable, his incentive constraint for this deviation is

$$-k(\mu) \leq 1 - m_1 - k(\mu). \tag{2}$$

Combining the two incentive constraints, we see that a value of m_1 exists to satisfy them both if and only if

$$\frac{1-\delta}{\delta} \leq \mu \left(\frac{1-\beta}{\beta} \right). \tag{3}$$

If a planner concerned with the players' welfare could manipulate the monitoring parameter μ and be sure that they would select such a grim trigger equilibrium in the resulting game, then (assuming $\beta \geq \delta$) the optimal choice would be $\mu^*_{\text{PPE}} = \frac{1-\delta}{\delta} \frac{\beta}{1-\beta}$ in order to encourage high effort at minimal monitoring cost.

1.2 Contractual equilibrium with fixed monitoring technology

Suppose the monitoring technology μ is fixed exogenously, so in the negotiation phase the players have only their immediate transfer and their self-enforced continuation play to discuss. They are endowed with fixed bargaining weights, given by $\pi_1 \geq 0$ and $\pi_2 \geq 0$ and satisfying $\pi_1 + \pi_2 = 1$, that determine how to divide any surplus they can obtain by agreeing, compared to disagreeing. If they disagree, then there is no immediate transfer, and they coordination on some continuation play from the action phase, anticipating that they will be able to agree in subsequent periods. Under disagreement, the worker's effort may be high or low, depending on the history. In contrast, if they agree then they maximize the sum of their payoffs subject to the equilibrium constraints, and use their immediate transfer to split the surplus relative to disagreement in proportion to their bargaining weights.

Since the environment is stationary, it follows that the players always earn the same sum of payoffs under agreement; let L denote this "joint value." Therefore the convex hull of the agreement payoff vectors they can obtain—the *value set*, denoted V—is a line segment of slope -1; moreover V contains its endpoints. Each endpoint of V is the payoff vector that arises from a bargaining problem whose disagreement point is achieved by incentive-compatible play in the current period followed by a continuation values selected from V as a function of the realized signal and the outcome of the public randomization device in the current period.

Let z^1 and z^2 be the endpoints of V, where z^1 is the worst continuation value for player 1 and z^2 is the worst for player 2. We can determine these endpoints using a recursive formulation, where we fix the line segment from z^1 to z^2 as the feasible continuation values from the next period and then we calculate the extremal continuation values $z^{1\prime}$ and $z^{2\prime}$ that can be supported from the start of the current period. The environment being stationary, we know that $z^{1\prime} = z^1$ and $z^{2\prime} = z^2$ for contractual equilibria (V must be self generating).

The disagreement point that achieves the extremal value that is worst for player 1 from the current period is characterized as follows: With no transfer, the players coordinate on $a = 1$ being played in the current period. Then, if the signal realization is $x = 1$, the players coordinate on behavior to achieve continuation value $z^1 + (\rho, -\rho)$. If the signal realization is $x = 0$ then the players coordinate on z^1 from the next period. The value of ρ must be large enough to ensure that the worker does not want to deviate to low effort:

$$-\beta(1-\delta) + \delta(z_1^1 + \rho) \;\geq\; (1-\delta)\cdot 0 + \mu\delta z_1^1 + (1-\mu)\delta(z_1^1 + \rho),$$

knowing that if she does deviate then with probability μ her deviation will be detected and she will be punished. Her incentive constraint simplifies to $\mu\delta\rho \geq \beta(1-\delta)$. It is optimal to pick the smallest possible value of ρ because player 1's expected payoff is increasing in ρ. So we can set

$$\rho = \frac{1-\delta}{\delta} \cdot \frac{\beta}{\mu}$$

and the disagreement value from the current period is

$$\underline{w}^1 = (1-\delta)(-\beta, 1-k(\mu)) + \delta z^1 + \delta(\rho, -\rho). \tag{4}$$

The players can renegotiate from this disagreement point, but it is already efficient so there is no surplus to negotiate over; so we have

$$z^{1\prime} = \underline{w}^1. \tag{5}$$

The disagreement-point that achieves the extremal value that is worst for player 2 from the current period is characterized as follows: With no transfer, the players coordinate on $a = 0$ being played in the current period and, regardless of the signal realization, the players coordinate on behavior to achieve continuation value z^2. Thus, the disagreement value from

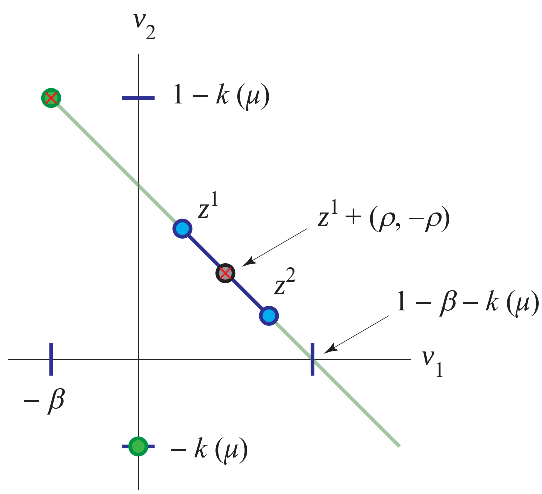

FIGURE 1. CONTRACTUAL EQUILIBRIUM WITH FIXED MONITORING TECHNOLOGY: CONSTRUCTION OF z^1.

the current period is

$$\underline{w}^2 = (1-\delta)(0, -k(\mu)) + \delta z^2. \tag{6}$$

The players negotiate from this disagreement point to obtain joint continuation value L, and they split the surplus according to their bargaining weights π, so we have

$$z^{2\prime} = \underline{w}^2 + \pi(L - \underline{w}^2_1 - \underline{w}^2_2). \tag{7}$$

To complete the calculations, we set $z^{1\prime} = z^1$ and $z^{2\prime} = z^2$, and we also observe that $L = z^1_1 + z^1_2 = z^2_1 + z^2_2$ must hold. Making these substitutions and simplifying expressions yields

$$z^1 = \left(\beta \cdot \frac{1-\mu}{\mu},\, 1 - k(\mu) - \frac{\beta}{\mu}\right)$$

and

$$z^2 = (0, -k(\mu)) + \pi(1-\beta).$$

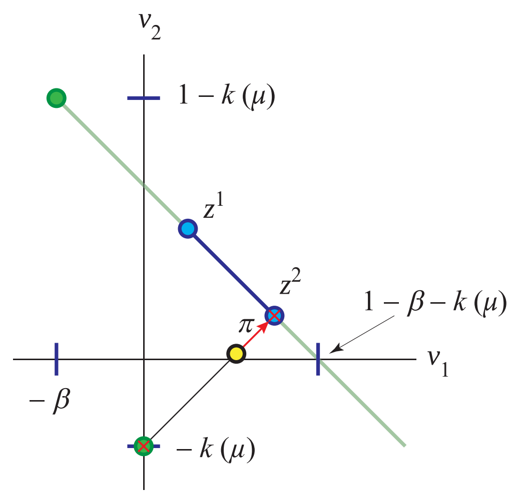

FIGURE 2. CONTRACTUAL EQUILIBRIUM WITH FIXED MONITORING TECHNOLOGY: CONSTRUCTION OF z^2.

Note that the *span* of the continuation-value line segment is

$$d \equiv z_1^2 - z_1^1 = z_2^1 - z_2^2 = \pi_1(1-\beta) - \beta \cdot \frac{1-\mu}{\mu}$$

and the level is

$$L = 1 - \beta - k(\mu).$$

This equilibrium outcome requires that $\rho \leq d$—i.e., the bonus the worker receives when the good outcome arises must not exceed the span of V—which we assumed in the derivation of z^1. Recalling that $\rho = \beta(1-\delta)/\delta\mu$, this condition can be expressed in terms of primitives as

$$\frac{1-\delta}{\delta} \leq \mu\pi_1\left(\frac{1-\beta}{\beta}\right) + (1-\mu)(-1). \tag{8}$$

If this inequality does not hold, then high effort cannot be sustained and the contractual-equilibrium value is $(0, -k(\mu))$.

It is important to note how the span and level depend on the monitoring technology μ. Observe that the span is increasing in μ, because with better monitoring the worker's reward ρ for signal H does not need to be as large to induce high effort, which allows z_1^1 to be reduced. Observe that the level is decreasing in μ because better monitoring costs more. The joint-value maximizing monitoring technology μ^* solves the problem of minimizing $k(\mu)$ subject to Eq. (8), which can be written:

$$\mu^* = \frac{1}{\pi_1}\left[\frac{1-\delta}{\delta}\cdot\frac{\beta}{1-\beta} + (1-\mu^*)\frac{\beta}{1-\beta}\right].$$

1.3 Contractual equilibrium with contractible monitoring technology

Now let us consider contracting on both the self-enforced and externally enforced elements of the relationship. The externally enforced component of contract specifies a sequence of monitoring technologies, $\{\mu^t\}$, where μ^t is the level of monitoring to be provided in period t. Assume that the default provision at the beginning of the game is $\mu^t = 0$ for all t. If the players make an agreement (which will be the case in each period in contractual equilibrium) then the sequence of monitoring levels that they select goes into effect and becomes the default externally enforced contractual element until they successfully renegotiate.

Because the set of feasible contracts is unchanged from period to period, and because contractual equilibrium predicts that the players will always bargain up to the maximal achievable joint continuation value, there is an equilibrium level L^* that is the joint continuation value from the start of every period, regardless of the history of play. However, the set of attainable continuation values from the beginning of a given period depends on the default externally enforced contract that is inherited from the most recent prior agreement. Therefore the endpoints z^1 and z^2 of the value set V depend on the terms of the outstanding externally enforced contract.

It turns out that, in a contractual equilibrium, stationary externally enforced contractual terms (specifying the same μ in all periods) are generally suboptimal. However, the optimal externally enforced terms are *semi-stationary* in that they specify one monitoring level $\hat{\mu}$ for the current period and another level $\tilde{\mu}$ for all future periods. In equilibrium, in each period the players revise their contract to specify $\hat{\mu}$ for the current period and retain $\tilde{\mu}$ for future periods.

Intuition gleaned from the fixed-μ case explains this result. To achieve the highest joint value in the current period, the players want μ in this period to be low to save on the monitoring cost. In order to support high effort with a low monitoring level in the current period, the players need the span of continuation values from the next period to be large. To maximize the span, it is best to specify a large monitoring level for future periods, which supports wide-ranging disagreement points even though the players anticipate renegotiating to a lower monitoring level in the next period.

Formally, Equation 4, Equation 6, and Equation 7 are valid for the setting in which the players contract on a sequence of monitoring levels, except that (i) we have L^* in place of L, (ii) we recognize that z^1 and z^2 depend on the externally enforced contractual elements in place at the beginning of the next period, and (iii) the monitoring level μ in the expressions is what is in force for the current period (not necessarily what is specified for future periods). In place of Equation 5, we add the following equation, recognizing that the players should renegotiate away from $\underline{w}^1$ if by doing so they can support high effort with monitoring costs lower than specified by the inherited contract:

$$z^{1\prime} = \underline{w}^1 + \pi(L - \underline{w}_1^1 - \underline{w}_2^1). \tag{9}$$

Let d be the span of the continuation-value set from period $t+1$ and let d' be the span achieved from the start of period t. That is,

$$d \equiv z_1^2 - z_1^1 = z_2^1 - z_2^2, \tag{10}$$

$$d' \equiv z_1^{2\prime} - z_1^{1\prime} = z_2^{1\prime} - z_2^{2\prime}. \tag{11}$$

Putting these six expressions (Equations 4, 6, 7, 9, 10, and 11) together and simplifying yields

$$d' = \delta d + (1-\delta)\left[\pi_1(1-\beta) - \beta \cdot \frac{1-\mu}{\mu}\right].$$

To maximize the span from the period t, it is clearly optimal to select $\mu = 1$.

Of course, when they negotiate in period t they will want to maximize the span not from period t, but from period $t+1$. Therefore they should agree on a contract that sets $\mu^{t+1} = 1$, regardless of the monitoring levels they choose for periods $t+2, t+3, \ldots$. Note as well that d' is increasing in d, so by induction the players want $\mu^{t+1} = \mu^{t+2} = \cdots = 1$. This means that the span from period $t+1$ is that which solves $d = d'$ with the above expressions, which is $d = \pi_1(1-\beta)$.

As for the current period t, to save on monitoring costs that they will actually have to pay today, the players optimally select the lowest monitoring level that can enforce the worker's high effort. This is the monitoring level for which the required H-signal bonus, $\rho = \beta(1-\delta)/\delta\mu$, just fits within the span d. Thus, the best choice of μ^t is the smallest value that satisfies these constraints, which is

$$\hat{\mu} = \frac{1}{\pi_1} \cdot \frac{1-\delta}{\delta} \cdot \frac{\beta}{1-\beta}.$$

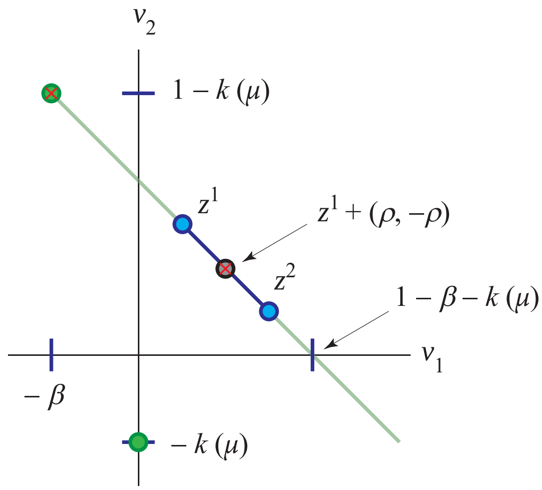

FIGURE 3. CONTRACTUAL EQUILIBRIUM WITH CONTRACTIBLE MONITORING TECHNOLOGY.

Note that $\hat{\mu} < \mu^*$ so the players get a strictly higher joint value using a semistationary contract than they would obtain with a stationary contract that has the same μ^* in every period.

To summarize, in the contractual equilibrium, in the first period the players choose externally enforced terms $\{\mu^t\}$ with $\mu^1 = \hat{\mu}$ and $\mu^t = 1$ for $t = 2, 3, \ldots$. In the second period, the players revise their contract by specifying $\mu^2 = \hat{\mu}$ and leaving the other externally enforced terms the same. Likewise, they renegotiate μ^3 in period 3, they renegotiate μ^4 in period 4, and so on.

2 The Model

We work with a hybrid model as in Miller and Watson (2013) with an external enforcement technology. Two players interact in discrete periods over an infinite horizon with discount factor δ. In each period, there are two phases: the cooperative *negotiation phase* where the players make a joint decision to form or revise their contract and make immediate monetary transfers, and the non-cooperative *action phase* where the players select individual actions and receive payoffs in a stage game. The stage game, which may vary from period to period, is compelled by the external enforcer as directed by the players' contract. We call this externally enforced part of the contract the *external contract*. Likewise, we will call the self-enforced aspects of the relationship the *internal contract*. At the end of each period there is also a draw from a public randomization device that we assume is uniformly distributed on the unit interval. We normalize stage-game payoffs by multiplying by $1-\delta$ as is standard.

The first subsection below describes the game, which includes the external enforcement technology. The second subsection describes how we specify the generalized strategies for the game, which we call *regimes*, that include the joint decisions and individual actions. The third subsection defines the contractual-equilibrium solution concept, which combines individual rationality (self-enforcement) and a theory of bargaining over both the self-enforced and externally enforced components of contract. Bargaining is resolved according to the generalized Nash bargaining solution, with fixed bargaining weights that represent in reduced form the exogenous parameters of a noncoopeative bargaining protocol.

2.1 Technology and external enforcement

Let us describe first the technological details of the relationship, including the scope for external enforcement. A stage game has the following components:

- a set of action profiles $A = A_1 \times A_2$,
- an outcome set X,
- a conditional distribution function $\lambda\colon A \to \Delta X$,
- a payoff function $u\colon A \times X \to \mathbb{R}^2$, and
- a partition P of X.

In the third bullet point, ΔX denotes the set of probability distributions over X. We write $a_i \in A_i$ as player i's individual action in the stage game. The function λ gives the distribution over X for a given action profile. That is, $\lambda(a)$ is the distribution of outcomes in the event that the players select $a \in A$. The outcome $x \in X$ is commonly observed by the players, so each player i knows x and his choice a_i. Player i observes nothing else about actions in the stage game.[11] The partition P represents the external enforcer's verifiability constraints with respect to the stage-game outcome, so that the enforcer can verify only the partition element $P(x)$ containing the realized outcome x.

[11] In some applications, it will be the case that player i's payoff $u_i(a,x)$ is a function of only x and player i's action $a_i \in A_i$, so that player i obtains no additional information about the other player's actions through player i's realized payoff, above what is already learned by observing the outcome x. For other applications, we will assume that $u_i(a,x)$ depends on the other players' actions but that player i does not observe his own payoff.

Turning to the interaction across periods, fundamentals of the model include a set G of feasible stage games, an abstract set of *external contracts* C, an initial external contract $\underline{c} \in C$, and a function $g : C \to G$. These elements describe external enforcement in a convenient recursive formulation, where the external contracts may be thought of as "continuation contracts." In a given period, an external contract $c \in C$ will be in effect, and $g(c) = (A, X, \lambda, u, P)$ gives the prescribed stage game that the external enforcer compels the parties to play in this period. To make the dependence on c clear, we sometimes write $A(c)$, $X(c)$, $\lambda(\,\cdot\,; c)$, $u(\,\cdot\,; c)$, and $P(\,\cdot\,; c)$ as the components of stage game $g(c)$. Further, define the set of feasible (c, x) pairs as

$$\mathbb{CX} \equiv \{(c, x) \mid c \in C \text{ and } x \in X(c)\}.$$

A transition function $\zeta \colon \mathbb{CX} \times [0, 1] \to C$ determines the external contract to be in effect at the beginning of the next period as a function of the current period's external contract, the outcome of the stage game in the current period, and the realization of the public randomization device in the current period. That is, if in the current period, the external contract is c, the outcome of the stage game is $x \in X(c)$, and the random draw is $\phi \in [0, 1]$, then $\hat{c} = \zeta(c, x, \phi)$ is the external contract in effect at the beginning of the next period. We call $\hat{c}$ the *inherited external contract* for the next period. To represent the external enforcer's verification constraints, the function ζ must be measurable with respect to the partition P for each stage game in G. This means that, for an external contract c, random draw $\phi \in [0, 1]$, and any two outcomes $x, x' \in X(c)$ that are in the same partition element (i.e., $x' \in P(x; c)$), we have $\zeta(c, x, \phi) = \zeta(c, x', \phi)$.

In summary, the contractual setting is defined by a set G of feasible stage games, a set C of external contracts with initial element $\underline{c} \in C$, a function $g : C \to G$, a transition function $\zeta \colon \mathbb{CX} \times [0, 1] \to C$, and the discount factor δ. At this point, we do not need to put any additional structure on the external enforcement technology. But to get a feel for the formulation, it may be useful to review some examples.

Consider, as an illustration, an external enforcement technology that allows for arbitrary transitions between stage games as a function of the verifiable outcome. This means that the external contract space is equivalent to the space of functions that map verifiable histories to the set of stage games. A verifiable history (what the external enforcer observes) from period 1 to any given period T is a sequence $\psi = \{\gamma^t, x^t, \phi^t\}_{t=1}^{T}$, where $\gamma^t = (A^t, X^t, \lambda^t, u^t, P^t) \in G$ denotes the stage game compelled in period t, $x^t \in X^t$ is the outcome, and $\phi^t \in [0, 1]$ is the draw of the public randomization device. Let Ψ be the set of all such feasible finite verifiable histories, where the case of $T = 0$ is included to denote the null verifiable history at the beginning of period 1. Then in this example, an external contract may be defined as any mapping c from Ψ to G that is measurable with respect to the enforcer's information partitions and the randomization device.[12]

Other examples of enforcement technologies include ones that recognize only a finite partition of the possible draws of the public randomization device; for these, the transition function ζ is assumed measurable with respect to this partition. One could also imagine enforcement technologies that require the same stage game in perpetuity after some bounded number of periods. Putting these two restrictions together with the assumption that G is finite would imply that the external contract space C is finite.

For the analysis and technical results, we shall assume throughout the paper that every stage game has a finite action space. That is, for every $(A, X, \lambda, u, P) \in G$, the set A is finite. This assumption is not required for our definitions; it is used in the arguments that establish existence of our solution concept, both in the next section and in the Appendix. We make some comments here and there about generalizing the model.

2.2 The relational contracting game

We can now describe the contracting game. In each period $t = 1, 2, \ldots$, play proceeds as follows. Players enter the period with an external contract $\hat{c}^t$ that is inherited from the previous period. In the case of $t = 1$, we assume $\hat{c}^1 = \underline{c}$. In the negotiation phase, the players negotiate to select an external contract $c^t \in C$ and an immediate monetary transfer $m^t \in \mathbb{R}^2_0$, where $\mathbb{R}^2_0 \equiv \{m \in \mathbb{R}^2 \mid m_1 + m_2 = 0\}$ is the set of real vectors whose components sum to zero (balanced transfers). The negotiated transfer is enforced automatically with the agreement in the sense that it "seals the deal."[13] If the players do not reach an agreement, then $c^t = \hat{c}^t$ and the transfer is zero. In the action phase of period t, the players simultaneously choose individual actions in stage game $g(c^t)$, outcome $x^t \in X(c^t)$ occurs, and the draw ϕ^t of the public randomization device is realized. Then the external contract inherited in period $t+1$ is $\hat{c}^{t+1} = \zeta(c^t, x^t, \phi^t)$.

The payoffs within a period are given by the sum of any monetary transfer and the stage-game payoffs, normalized by $1 - \delta$. That is, if the players transfer $m \in \mathbb{R}^2_0$, play action profile a in stage game (A, X, λ, u, P), and get outcome $x \in X$, then the payoff vector

[12]For any stage game $\gamma = (A, X, \lambda, u, P)$, outcome $x \in X$, and public draw ϕ, and for any T-period verifiable history ψ, let "$\psi^\frown(\gamma, x, \phi)$" denote the sequence formed by concatenating ψ and (γ, x, ϕ). The transition function ζ is defined so that $\zeta(c, x, \phi) = c(\psi^\frown(g(c), x, \phi))$ for all $\psi \in \Psi$.

[13]See Miller and Watson (2013) for the non-cooperative details.

for this period is $(1-\delta)(m+u(a,x))$. As the game progresses, the players' behavior (joint actions and individual actions), along with the outcomes of the exogenous random variables, induces a sequence $\{m^t, u^t, a^t, x^t\}_{t=1}^{\infty}$. The realized continuation payoff vector from any period τ is then

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau}(1-\delta)\left(m^t + u^t(a^t, x^t)\right). \tag{12}$$

Because the players may randomize in their choice of actions and there is exogenous uncertainty in the outcomes of the stage game and randomization devices, the sequence $\{m^t, u^t, a^t, x^t\}$ will be random and so the continuation payoff vector is given by the expectation of Expression 12, conditioned on the history prior to time τ and the players' equilibrium.

2.3 Regimes and continuation values

We next define a generalized notion of strategy, which we call a *regime*, to represent the specification of both individual actions in the action phases and joint decisions in the negotiation phases. To define a regime, we first must set some notation for histories.

A shared (public) T-period history for the players is a sequence $\{(c^t, m^t, x^t, \phi^t)\}_{t=1}^{T}$ with the property that $x^t \in X(c^t)$ for each $t \in \{1, 2, \ldots, T\}$. Here c^t is the external contract and m^t is the transfer jointly chosen by the players in period t. For $t > 1$, if c^t does not equal the inherited external contract $\zeta(c^{t-1}, x^{t-1}, \phi^{t-1})$, then it means that the players renegotiated in period t to change their external contract.[14] The stage-game outcome x^t and ϕ^t are commonly observed by the players and thus included in the history. The individual actions a^t are not included because the players do not commonly observe these private actions. In our solution concept, which is of the standard "public equilibrium" variety, the joint and individual actions on the equilibrium path will be a function of only the commonly observed outcome variables. Let H be the set of all finite histories, including the initial (null) history h^0.

A regime $r = (r^{\mathrm{c}}, r^{\mathrm{m}}, r^{\mathrm{a}})$ describes the prescribed joint decision and individual actions as a function of the history. The function $r^{\mathrm{c}} \colon H \to C$ gives the specified choice of the external contract at the beginning of each period, as a function of the history. The func-

[14]This accounting of histories does not differentiate between disagreement and agreeing to keep the contractual arrangements unchanged and to make no transfer. Both would be represented by $c^t = \zeta(c^{t-1}, x^{t-1}, \phi^{t-1})$ and $m^t = 0$. It turns out that the analysis of the model is not affected by whether this distinction is made, and it is simpler to go without it.

tion $r^{\mathrm{m}}\colon H \to \mathbb{R}_0^2$ gives the associated immediate transfer that the players agree to. That is, $(r^{\mathrm{c}}(h), r^{\mathrm{m}}(h))$ is the prescribed joint decision in the negotiation phase of the period following history h. Finally, the function $r^{\mathrm{a}}\colon H \times C \times \mathbb{R}_0^2 \to \cup_{c \in C} \Delta A(c)$ gives the mixed action profile as a function of the history to the action phase in any period. That is, if following history h the players jointly choose (c, m) in the current period, then the prescribed action profile for the current period is $r^{\mathrm{a}}(h, c, m) \in \Delta A(c)$. Because we assume that the players randomize independently, $\Delta A(c)$ is taken to mean the uncorrelated distributions over $A(c)$.

For a given regime r and any history $h \in H$, let $v(h; r)$ be the vector of expected continuation values from the beginning of the period following history h, assuming that the players behave as specified by r from this point in the game. Likewise, let $y(h, c, m, a; r)$ be the expected continuation value following history h under the assumption that the players jointly select (c, m) in the negotiation phase of the current period, the individual action profile is $a \in A(c)$ in the current period, and the players behave as specified by r in all future periods. Thus, for any $h \in H$, $c \in C$, $m \in \mathbb{R}_0^2$, and $a \in A(c)$, we have:

$$y(h, c, m, a; r) = (1 - \delta)m + E_{x,\phi}\left[(1 - \delta)u(a, x; c) + \delta v(h, (c, m, x, \phi); r)\right],$$

where we recall that $u(\cdot; c)$ is the payoff function for stage game $g(c)$, and the expectation is taken with respect to $x \sim \lambda(a; c)$ and $\phi \sim U[0, 1]$. Also, we have

$$v(h; r) = E_a\left[\, y(h, r^{\mathrm{c}}(h), r^{\mathrm{m}}(h), a; r)\,\right],$$

where the expectation is taken with respect to $a \sim r^{\mathrm{a}}(h, r^{\mathrm{c}}(h), r^{\mathrm{m}}(h))$.

For a T-period history $h \in H$, let us denote by $\hat{c}(h) = \zeta(c^T, x^T, \phi^T)$ the external contract inherited in period $T + 1$ following history h. Finally, for any history $h \in H$ and a regime r, let $\underline{v}(h; r)$ denote the *disagreement point* for the bargaining phase in the period following history h. This is the continuation value under the assumption that the players fail to reach an agreement in the current period, and thus $c^t = \hat{c}(h)$ and $m = 0$, but play in the action phase of the current period and all future behavior is specified by the regime r. That is,

$$\underline{v}(h; r) = E_a\left[\, y(h, \hat{c}(h), 0, a; r)\,\right],$$

where the expectation is take with respect to $a \sim r^{\mathrm{a}}(h, \hat{c}(h), 0)$.

2.4 Contractual equilibrium

Contractual equilibrium combines two conditions. First, we have the standard sequential rationality condition for individual actions. In the action phase of each period, the players best-respond to each others' actions, given their anticipated behavior in the continuation of the game.

Definition 1. *A regime r is called* **incentive compatible in the action phase** *if for all $h \in H$, $c \in C$, $m \in \mathbb{R}^2_0$, and for each player i and action $a'_i \in A_i(c)$, it is the case that $y(h, c, m, r^{\mathrm{a}}(h, c, m); r) \geq y(h, c, m, (a'_i, r^{\mathrm{a}}_{-i}(h, c, m)); r)$. That is, player i cannot gain by deviating from $r^{\mathrm{a}}_i(h, c, m)$ in the action phase following history h and joint decision (c, m) in the current period.*

The second condition is that, in each period, the players' joint action in the negotiation phase is characterized by the generalized Nash bargaining solution, with fixed bargaining weights given by $\pi \in \mathbb{R}^2$ where $\pi_i \geq 0$ for all i and $\pi_1 + \pi_2 = 1$. That is, the players reach an agreement that maximizes their joint value and they split the bargaining surplus according to their bargaining weights. Note that π is a parameter of the bargaining solution; it summarizes in reduced formthe parameters of a corresponding noncooperative bargaining protocol. Miller and Watson (2013) and Watson (2013) provide non-cooperative foundations.

Importantly, we assume that the players negotiate over both the external part of the contract and the internal, self-enforced part. The former amounts to the selection of c and an immediate transfer. The latter means coordinating on a regime for the continuation of the game, which includes individual actions in the current and future periods as well as anticipated joint decisions in future periods (all as a function of the history). Following Miller and Watson (2013), we capture this condition by first imposing an *internal consistency* agreement condition, which represents the following idea: In equilibrium, the players recognize that, after any history $h \in H$, they have the option of agreeing to continue as though the history is any other $h' \in H$. That is, the players have the option of selecting the external contract that they would have selected following h' and then plan to play as their regime specifies from history h'.[15] Since the players can make any transfer in the negotiation phase, they are able to split the negotiation surplus in any way desired, and our bargaining assumption implies that they split the surplus according to π.

[15]Note that this is feasible because, just after selecting $r^{\mathrm{c}}(h')$ and any transfer, the continuation game would be the same as from the action phase following h'.

Definition 2. *A regime r is called* **internally bargain-consistent** *if for all $h \in H$,*

$$v(h;r) = \underline{v}(h;r) + \pi \max_{h' \in H} \left(v_1(h';r) + v_2(h';r) - \underline{v}_1(h;r) - \underline{v}_2(h;r) \right).$$

The following lemma is derived straight from the definition of internal consistency.

Lemma 1: *If regime r is internally bargain-consistent, then it has the same joint value from the beginning of any period. That is, there exists $L \in \mathbb{R}$ such that $v_1(h;r) + v_2(h;r) = L$ for all $h \in H$.*

For a regime that is internally consistent, let us call L its *joint value* or just *level*. The players jointly prefer to coordinate on a regime that maximizes L, and this condition completes the definition of contractual equilibrium:

Definition 3. *Given bargaining weights π, a regime is called a* **contractual equilibrium (CE)** *if it is incentive compatible in the action phase and internally bargain-consistent, and its level is maximal among the set of regimes with these properties.*

We have the following obvious implication of the contractual-equilibrium definition.

Lemma 2. *For a given relational-contract setting, all contractual equilibria attain the same level.*

3 Optimal Contracts and Semi-Stationarity

In a contractual equilibrium, the players negotiate in each period to select external and internal contract terms that maximize their joint value, accounting for the fact that the regime will be renegotiated when the next period starts. In principle, the optimal external contract may be complicated, specifying different stage games after different histories in order to punish and reward the players for on their past behavior. However, we show in this section that simple contracts are optimal in a broad range of settings.

3.1 Noncontingent and semi-stationary contracts

Let us consider three categories of simple external contracts. The first category contains external contracts that specify a fixed sequence of stage games, independent of the verifiable history of play. The second and third categories are subsets that specify the same stage game over time, regardless of the history.

Definition 4. *An external contract $c \in C$ is called* **noncontingent** *if there is a sequence $\{c^k\} \subset C$ such that $c^1 = c$ and, for every $k = 1, 2, \ldots$, the transition function satisfies $\zeta(c^k, x, \phi) = c^{k+1}$ for all $x \in X(c^k)$ and $\phi \in [0, 1]$.*

Definition 5. *An external contract $c \in C$ is called* **stationary** *if $\zeta(c, x, \phi) = c$ for all $x \in X(c)$ and $\phi \in [0, 1]$.*

Note that, since a stationary external contract c always transitions back to itself, it specifies stage game $g(c)$ in every period regardless of the history. Note that this form of stationarity pertains to only the externally contract; the internal, self-enforced terms that the players' regime specifies may change over time and be sensitive to the history.

Definition 6. *An external contract $c \in C$ is called* **semi-stationary** *if there is a stationary external contract $\underline{c}$ such that $\zeta(c, x, \phi) = \underline{c}$ for all $x \in X(c)$ and $\phi \in [0, 1]$. In this case, say that c* **transitions to** *$\underline{c}$.*

A semi-stationary external contract starts with stage game $g(c)$ and specifies stage game $g(\underline{c})$ in all future periods regardless of the history.

Definition 7. *A regime $r = (r^{\mathrm{c}}, r^{\mathrm{m}}, r^{\mathrm{a}})$ is called* **semi-stationary** *if there is a semi-stationary external contract c such that $r^{\mathrm{c}}(h) = c$ for all $h \in H$.*

In a semi-stationary regime, the players always negotiate to the same semi-stationary external contract c that would transition to some stationary external contract $\underline{c}$ absent negotiation. This means that, although the external contract selects stage game $g(c)$ in the current period and $g(\underline{c})$ in all future periods, in each period the players negotiate back to stage game $g(c)$ in the current period and they postpone reversion to $g(\underline{c})$.

As we show in the next two subsections, in a wide range of contractual settings it is optimal for the players to select semi-stationary external contracts. To be more precise, there are semi-stationary contractual equilibria. Some technical conditions are required for the result, starting with an assumption that C contains all possible semi-stationary external contracts:

Assumption 1. *For every pair of stage games $\gamma, \underline{\gamma} \in G$, there is a semi-stationary external contract $c \in C$ that transitions to some stationary contract $\underline{c} \in C$ with the property that $g(c) = \gamma$ and $g(\underline{c}) = \underline{\gamma}$.*

The enforcement technology that allows for arbitrary selection of the stage game as a function of the verifiable history (the setting described at the end of Section 2.1) satisfies this assumption.

3.2 Semi-stationary with transfers

In many settings the external enforcer can compel arbitrary transfers as a function of the enforcer's verifiable information about the stage-game outcome.

Definition 8. *The contractual setting has* **externally enforced transfers** *if for every $c \in C$ and every $P(\,\cdot\,; c)$-measurable function $\beta : X \to \mathbb{R}^2_0$, we have $(A, X, \lambda, u + \beta, P) \in G$ as well.*

To understand this definition, think of β as specifying a monetary transfer between the players as a function of the verifiable outcome $x \in X$. Transforming a stage game by changing the payoff function from u to $u + \beta$ is equivalent to adding an externally enforced transfer, which is feasible because β is $P(\,\cdot\,; c)$-measurable.

Our main result is that, under some technical conditions sufficient for existence, semi-stationary contracts are optimal in settings with externally enforced transfers. Let us start by developing intuition and the basic conceptual argument. We will use some recursive techniques that relate rational behavior in one period to sets of continuation values from the start of the next period.

The main logic

For any external contract c, let $W(c)$ be the set of continuation values that can be supported in a contractual equilibrium from the start of a period with inherited contract c. Assume for now that $W(c)$ is nonempty. Because all contractual-equilibrium continuation values (from the beginning of periods) attain the same level L, we know that $W(c)$ is a subset of the line $\{w \in \mathbb{R}^2 \mid w_1 + w_2 = L\}$ for every $c \in C$. Let us presume that every set $W(c)$ is bounded and thus has a finite *span*, which is defined as the horizontal (equivalently, vertical) distance between the endpoints of the set. Further, suppose that the level of $W(c)$ is maximized by some external contract $\tilde{c}$ and that $W(\tilde{c})$ contains its endpoints z^1 and z^2, where z^1 denotes the value that is worst for player 1 and z^2 is the worst value for player 2. Thus, the span of $W(\tilde{c})$ is $z^2_1 - z^1_1 = z^1_2 - z^2_2$.

Suppose that, following a given history, the players would optimally select external contract c^t which requires them to play stage game $g(c^t) = (A, X, \lambda, u, P)$ in the current period t. Incentives in the action phase are influenced both by the stage-game payoffs of $g(c^t)$ and by the continuation value v^{t+1} starting in period $t+1$. Note that v^{t+1} is contingent on the outcome of the action phase in period t. Thus, we can write $v^{t+1} = z(x, \phi)$ for some function $z\colon X \times [0, 1] \to \mathbb{R}^2$. In the action phase of period t, the players are essentially playing the artificial game that has the space A of action profiles and payoffs given by

$$U(a) \equiv E_{x,\phi}[\,(1-\delta)u(a,x) + \delta z(x,\phi)\,], \tag{13}$$

for each $a \in A$, where the expectation is taken with respect to $x \sim \lambda(a)$ and $\phi \sim U[0,1]$. Let us examine how we can transform u and z *without altering this artificial game*.

Clearly we must have $z(x,\phi) \in W(\zeta(c^t,x,\phi))$ for all $x \in X$ and $\phi \in [0,1]$. That is, $z(x,\phi)$ must be a contractual-equilibrium continuation value associated with the external contract inherited in period $t+1$. Because $W(\tilde{c})$ has the greatest span, we can find a function $\eta\colon X \times [0,1] \to \mathbb{R}^2_0$, representing transfers between the players, such that

$$z(x,\phi) - \eta(x,\phi) \in W(\zeta(c^t,x,\phi)) - \eta(x,\phi) \subset \operatorname{Conv} W(\tilde{c})$$

for all $x \in X$ and $\phi \in [0,1]$. Here "Conv" denotes the convex hull. Further, taking the expectation over ϕ, we have

$$E_\phi[z(x,\phi) - \eta(x,\phi)] \in \operatorname{Conv} W(\tilde{c})$$

for all x. We can then find a function $z'\colon X \times [0,1] \to \mathbb{R}^2$ with the properties $z'(x,\phi) \in \{z^1, z^2\}$ for all x and ϕ, and

$$E_\phi[z'(x,\phi)] = E_\phi[z(x,\phi) - \eta(x,\phi)]$$

for every x. That is, to achieve the specified expected continuation value in $\operatorname{Conv} W(\tilde{c})$ for any particular $x \in X$, we can randomize over the endpoints of $W(\tilde{c})$ using the public random draw ϕ to achieve the needed probabilities.

The foregoing analysis allows us to rewrite Equation 13 by substituting in the new function z', which maps into the set $\{z^1, z^2\}$:

$$U(a) = E_{x,\phi}[\,(1-\delta)u(a,x) + \delta\eta(x,\phi) + \delta z'(x,\phi)\,]. \tag{14}$$

The final step is to define a function $\beta\colon X \to \mathbb{R}^2_0$ by setting $\beta(x) = E_\phi[\frac{\delta}{1-\delta}\eta(x,\phi)]$ for all $x \in X$. Then we can define payoff function u' by $u'(a,x) = u(a,x) + \beta(x)$ and, substituting for this in Equation 14, we obtain:

$$U(a) = E_{x,\phi}[\,(1-\delta)u(a,x) + \delta z(x,\phi)\,] = E_{x,\phi}[\,(1-\delta)u'(a,x) + \delta z'(x,\phi)\,]. \tag{15}$$

Note that replacing u with u' is possible because, from the assumption of externally enforced transfers, $(A, X, \lambda, u', P) \in G$.

To summarize, we transformed the stage game for period t by adding the transfer function β. Correspondingly, we changed the specification of continuation values in period $t+1$, now given by the function z' that maps to only the endpoints of $W(\tilde{c})$. In other words, whereas the original specification used a selection of various sets of continuation values $W(c)$ in period $t+1$ to motivate the individual actions in period t, we replaced this selection with transfers at the end of period t. This was possible because (i) continuation-value sets have the same level, so varying between them is equivalent to making a transfer; and (ii) both the externally enforced transfer at the end of period t and the selection of the inherited external contract in period $t+1$ are conditioned on the verifiable outcome of the stage game in period t. In the end, the effective game being played in the action phase of period t is unchanged, so we can support the same behavior and continuation values as we could originally.

In terms of external contract specifications, these adjustments would be accomplished by replacing the given external contract c^t with an external contract $c' \in C$ for which $g(c') = (A, X, \lambda, u', P)$ and $\zeta(c', x, \phi) = \tilde{c}$ for all $x \in X$ and $\phi \in [0, 1]$. Importantly, c' is noncontingent in the transition from the current period t to period $t+1$; that is, it specifies inherited external contract $\hat{c}^{t+1} = \tilde{c}$ regardless of the outcome of the action phase in period t.

We can repeat the argument with $\tilde{c}$ in period $t+1$. That is, we can find a way to change the stage game and selection of continuation values transition from period $t+1$ to period $t+2$ so that the transition from $\tilde{c}$ to the inherited external contract in period $t+2$ is noncontingent. Proceding by induction, we can identify a noncontingent contract that achieves the same continuation values as does c^t, the contract we started with.

Holding aside whether the noncontingent contract identified by this procedure is actually an element of C, we can go further by recognizing that a key step in the above logic is finding a set $W(\tilde{c})$ that has the largest span among sets of equilibrium continuation values. Because stage game $g(\tilde{c})$ specified in period $t+1$ is instrumental in achieving the largest span, we would expect that it would be useful to utilize it not just in period $t+1$ but in future periods as well. This is indeed the case, and it implies the optimality of a semi-stationary contract that specifies the same stage game in all periods $t+1, t+2, \ldots$. We next describe the necessary technical conditions and the main result.

Optimality result

Consider a given period with stage game $\gamma = (A, X, \lambda, u, P) \in G$ and let $z\colon X \times [0,1] \to \mathbb{R}^2$ specify the continuation value from the next period as a function of the outcome and public random draw in the current period. Let $\overline{u}^\gamma(a) \equiv E_x[\,u(a,x)\,]$ and let $\overline{z}(a) \equiv E_{x,\phi}[\,z(x,\phi)\,]$, where the first expectation is taken with respect to $x \sim \lambda(a)$ and the second expectation is take with respect to $x \sim \lambda(a)$ and $\phi \sim U[0,1]$. Also, we will write A^γ and X^γ as the sets of action profiles and outcomes in stage game γ. Then in the action phase of the current period, the players are essentially playing the artificial game $\langle\, A^\gamma, (1-\delta)\overline{u}^\gamma + \delta\overline{z}^\gamma \,\rangle$ and incentive compatibility is given by the following definition.

Definition 9. *Given $\gamma \in G$ and $z\colon X^\gamma \times [0,1] \to \mathbb{R}^2$, call action profile $\alpha \in \Delta A^\gamma$* **enforced** *(relative to γ and z) if it is a Nash equilibrium of $\langle\, A^\gamma, (1-\delta)\overline{u}^\gamma + \delta\overline{z}^\gamma \,\rangle$.*

We will characterize the span that can be generated for continuation values at the beginning of the current period. Because negotiation will lead to a constant level, we normalize the continuation values from the action phase so that they lie on the line $\mathbb{R}^2_0$ (zero joint value). The normalization is done by shifting stage-game payoffs along the ray π. This translates a payoff vector (u_1, u_2) to $(\pi_2 u_1 - \pi_1 u_2,\, \pi_1 u_2 - \pi_2 u_1)$. We also normalize continuation values from the next period to be on the line segment

$$\mathbb{R}^2_0(d) \equiv \{m \in \mathbb{R}^2 \mid m_1 + m_2 = 0 \text{ and } m_1 \in [0,d]\},$$

for a given span d.

We want to maximize the span of the induced set of continuation values from the current period (written below as the difference between player 1's best and worst continuation values) by choice of the stage game and action profiles. That is, we look for a stage game γ and action profiles α^1 and α^2, where α^1 supports a continuation value that is worst for player 1 and α^2 supports a continuation value that is best for player 1 (worst for player 2). These action profiles must be enforced relative to the stage game and some selection z^γ of continuation values from the start of the next period. For any stage game γ, action profile $\alpha \in \Delta A^\gamma$, and function $z\colon X \times [0,1] \to \mathbb{R}^2(r)$, define

$$\omega_1(\alpha, \gamma, z) = (1-\delta)(\pi_2 \overline{u}_1^\gamma(\alpha) - \pi_1 \overline{u}_2^\gamma(\alpha)) + \delta \overline{z}_1(\alpha).$$

This is player 1's normalized continuation value. Then let $\Lambda(d)$ denote the maximized difference across stage games and enforced action profiles:

$$
\begin{aligned}
\Lambda(d) \equiv \quad & \max && \omega_1(\alpha^2,\gamma,z) - \omega_1(\alpha^1,\gamma,z) \\
& \text{by choice of:} && \gamma \in G,\ z\colon X^\gamma \times [0,1] \to \mathbb{R}^2_0(d),\ \text{and } \alpha^1,\, \alpha^2 \in \Delta A^\gamma \\
& \text{subject to:} && \alpha^1 \text{ and } \alpha^2 \text{ are enforced relative to } \gamma \text{ and } z.
\end{aligned} \tag{16}
$$

A second optimization problem is to maximize the joint value attained in the current period, by choice of the stage game and action profile. Here as well, we normalize the continuation values.

$$
\begin{aligned}
\Xi(d) \equiv \quad & \max && \overline{u}_1^\gamma(\alpha) + \overline{u}_2^\gamma(\alpha) \\
& \text{by choice of:} && \gamma \in G,\ z\colon X^\gamma \times [0,1] \to \mathbb{R}^2_0(d),\ \text{and } \alpha \in \Delta A^\gamma \\
& \text{subject to:} && \alpha \text{ is enforced relative to } \gamma \text{ and } z.
\end{aligned} \tag{17}
$$

Our main result establishes that, assuming that Optimization Problems 16 and 17 have solutions, a contractual equilibrium exists and is semi-stationary.

Theorem 1. *Suppose Assumption 1 holds and the contractual setting has externally enforced transfers. If $\Lambda(d)$ and $\Xi(d)$ exist for all $d \geq 0$ then there exists a semi-stationary contractual equilibrium.*

Since by definition all contractual equilibria in the game attain the same joint payoffs, this theorem shows that semi-stationarity is optimal.

3.3 Semi-stationarity with no verifiable information

Next consider settings in which the external enforcer cannot distinguish between any stage-game outcomes.

Definition 10. *The contractual setting is said to have* **no verifiable information** *if for every $g = (A, X, \lambda, u, P) \in G$, the partition P is trivial: $P = \{X\}$.*

Without verifiable information, a contract specifies the sequence of stage games to be played but cannot make the sequence conditional on the history of stage-game outcomes. For instance, the example in Section 1 has no verifiable information, because the external enforcer does not observe the monitoring signal. The following result shows that semi-stationarity is optimal when there is no verifiable information, even if the external contracting authority will not compel transfers.

Theorem 2. *Suppose Assumption 1 holds and the contractual setting has no verifiable information. If* $\Lambda(d)$ *and* $\Xi(d)$ *exist for all* $d \geq 0$ *then there exists a semi-stationary contractual equilibrium.*

The proof of this theorem amounts to a slight variation of the steps establishing Theorem 1. For any relational contract setting, augment G so that there are externally enforced transfers. This will change neither the contractual-equilibrium set nor Optimization Problems 16 and 17, because externally enforced transfers cannot be conditioned on the outcome of the action phase in any period. In other words, externally enforced transfers can be only constants, which coincide with what the players can do voluntarily in the course of bargaining in each period. From Theorem 1, we have a semi-stationary contractual equilibrium. If such an equilibrium specifies selection of non-zero externally enforced transfers, it is straightforward to replace these transfers with voluntary transfers in the bargaining phase and the equilibrium conditions remain satisfied.

4 More examples

4.1 Partnership with choice of production technology

In addition to principal-agent settings, it is instructive to consider equal partnerships as well. Consider a partnership, in a contracting setting with no verifiable information but in which an external enforcer can impose a production technology. The partners have equal bargaining power (i.e., $\pi_1 = \pi_2 = \frac{1}{2}$), and each partner i either exerts high effort (i.e., plays $a_i = 1$) at cost β, or low effort (i.e., plays $a_i = 0$).

The basic technology entails a severe free-rider problem in that the benefits generated by efforts are equally shared among the partners, irrespective of who exerts effort; and in addition being cheated (by one's partner shirking) is worse when one doesn't cheat. We will see that in this case the unique contractual equilibrium entails low effort by both partners forever, no matter how patient they are. But suppose alternative technologies can be enforced, which improve the payoff for a partner who exerts high effort while his partner shirks, but at the cost of reducing the payoffs if both partners exert high effort. (This could be due to a technology that improves the productivity of individual effort, but requires extensive and costly coordination to be effective when employed by both partners.) We will see that the partners may then be able to obtain efficient payoffs if they are patient enough.[16]

[16]Alternatively, the model may represent two parties that are affected by pollution, but where each may do costly abatement activity. The abatement benefits are equally shared between the parties, irrespective of who

Specifically, suppose that for technology $T \in [0, \overline{T}]$ the revenue for each partner i is $1-T$ if both exerted high effort, $\sigma + T\xi$ if i exerted high effort while $-i$ exerted low effort, and σ if i exerted low effort while $-i$ exerted high effort, and zero if both exerted low effort. Assume that $\xi > 0$, so that when T is increased, the outcome improves for a partner who exerts high effort while the other does not, at the expense of both partners if they both exert high effort. These parameters generate the stage game

	C	D
C	$1-T-\beta, 1-T-\beta$	$\sigma+T\xi-\beta, \sigma$
D	$\sigma, \sigma+T\xi-\beta$	$0, 0$

Assume that $0 < 1-\beta < \sigma < \beta < 1$, so at $T = 0$ the stage game is a "submodular" prisoners' dilemma (being cheated is worse when one doesn't cheat). Assume that with technology $T = \overline{T}$ one partner's high effort is jointly as productive as both partners' high efforts (i.e., $2(1-\overline{T}-\beta) = 2\sigma + \overline{T}\xi - \beta$), and at $T = \overline{T}$ the joint payoffs from high effort are strictly positive (i.e., $1-\overline{T} > \beta$).

If the external enforcer is willing to impose only $T = 0$, Miller and Watson (2013) show that under these assumptions the unique contractual equilibrium outcome is for both partners to exert low effort forever, regardless of how patient they are. The problem is that asymmetric play under disagreement must involve action profiles $a = (0, 1)$ and $a = (1, 0)$, which when $\overline{T} = 0$ are so expensive to enforce that the necessary span of continuation values cannot be supported by the disagreement points they generate.

If instead the external enforcer is willing to impose any $T \in [0, \overline{T}]$ the partners choose, then there exist T' and T'' such that the stage game is a submodular prisoners' dilemma for $T \in [0, T']$, a "supermodular" prisoners' dilemma for $T \in (T', T'')$, and "chicken" for $T \in [T'', \overline{T}]$. We show that this capability enables the players to obtain efficient payoffs if they are patient enough.

First we construct an incentive compatible and internally bargain-consistent equilibrium in which the partners can support mutual cooperation $a = (1, 1)$ under $T = 0$ along the equilibrium path if they are patient enough. Let the external contract terms they write in each period specify $T = 0$ for the current period, with $T = \tilde{T} \in [T'', \overline{T}]$ for all future periods (i.e., such that at $T = \tilde{T}$ the stage game is chicken). Suppose they cooperate along the equilibrium path, so the level is $L = \frac{1}{1-\delta}(2-2\beta)$.

Consider a history off the equilibrium path, when player 1 is supposed to be punished

undertakes abatement. Technology T may then be thought of as reducing the cost of single-party abatement at the expense of increasing the cost of joint abatement, e.g. because the technology requires a scarce input that is more costly to use if more than one party uses it.

and the players have just disagreed. Then they play a chicken game with $T = \tilde{T}$ in the current period, and expect to renegotiate in the following period. Since $a = (1, 0)$ is a stage game equilibrium, it can be supported by a continuation value that does not change with the stage game outcome. In this case the internal contract terms can specify $a = (1, 0)$ followed by player 1's worst continuation payoff vector, z^1. This plan generates a disagreement continuation payoff vector of

$$y^1 = \big(\sigma + \tilde{T}\xi - \beta + \delta z_1^1, \sigma + \delta(L - z_1^1)\big). \tag{18}$$

Now step back to the start of the period. Knowing that y^1 is what they will get if they disagree, they renegotiate to the payoff vector z^1, characterized by:

$$z^1 = y^1 + \frac{1}{2}\big(L - y_1^1 - y_2^1, L - y_1^1 - y_2^1\big) \tag{19}$$

For $a = (1, 1)$ to be incentive compatible when they play the submodular prisoners' dilemma on the equilibrium path, it must be that $\frac{1}{2}L \geq \sigma + \delta z_1^1$. This is the case if $\delta < 1$ is sufficiently high:

$$\delta \geq \frac{2 - 2\beta - 2\sigma}{2 - 3\beta - 2\sigma + T\xi} < 1 \tag{20}$$

(note that both numerator and denominator are negative).

Intuitively, the chicken game they play after a disagreement has two pure strategy stage game equilibria that are Pareto incomparable. If a disagreement occurs, they coordinate to play the equilibrium that is relatively worse for whichever player is being punished. Knowing that this is what will happen if a disagreement occurs, when renegotiating they agree on a point in the equilibrium value set that is relatively worse for the player who is being punished; they implement this point by first having that player pay a transfer, and then continuing with efficient equilibrium path play of $a = (1, 1)$ in the stage game with $T = 0$. The size of the transfer is constant in average utility terms, which means the size of the transfer in total utility terms can be made arbitrarily large by choosing a sufficiently high discount factor. When they are patient enough the transfer suffices to enforce efficient equilibrium path play.

The players have the flexibility to choose T by writing an enforceable contract. By Theorem 1 they optimally choose a semi-stationary contract. From Eq. (20), conditional on $\tilde{T} \in [T'', \overline{T}]$, they can attain efficiency for the widest range of discount factors by specifying $\tilde{T} = T''$; i.e., minimize $\tilde{T}$ subject to playing Chicken under disagreement.. We show in the appendix that in fact $\tilde{T} = T''$ is the global optimum; i.e., they do not want the stage game to be a prisoners' dilemma under disagreement. Intuitively, asymmetric play in Chicken requires no incentives, allowing the continuation value to simply be z^1 under disagreement when partner 1 is being punished; whereas asymmetric play in the prisoners' dilemma requires incentives, so the continuation value must give partner 1 more than z_1^1 for playing $a_1 = 1$ when being punished under disagreement. As a result, further decreasing $\tilde{T}$ below T'' into the prisoners' dilemma range actually reduces the span of the regime.

4.2 Monitoring with managerial choice of technology.

In the previous analysis of monitoring we compared contractual equilibrium with and without external enforcement of technology, but where in the latter case the technology was exogenously given. Another relevant case is one where the manager can choose the technology and there is no external enforcement. We will now consider this case, and assume that in each period, the manager chooses μ and that this choice is observed by the worker before she chooses effort.

Given no external enforcement, both parties must be provided with self-enforced incentives (via continuation values) to take appropriate actions. Consider a contractual equilibrium –which we know is stationary – where the manager chooses monitor level μ and the worker exerts effort every period.

Recall that z^i is the continuation value that is worst for player i, $i = 1, 2$. In equilibrium incentives are provided as follows. The parties coordinate on continuation value $z^1 + (\rho, -\rho)$ if the monitor signal is high and μ was selected, on continuation value z^1 if the monitor signal is low and μ was selected, and on continuation value z^2 if the manager deviated from μ (irrespective of the signal). Given that μ is selected, the worker will exert effort as intended if

$$\delta\rho \geq (1-\delta)\beta/\mu. \tag{21}$$

If the manager deviates to some $\mu' \neq \mu$, the worker has no incentives for effort, hence the manager gets payoff $(1-\delta)(-k(\mu')) + \delta(z_2^1 - d)$. So the manager will not deviate if

$$(1-\delta)(1-k(\mu)) + \delta z_2^1 + \delta(-\rho) \geq (1-\delta)(-k(0)) + \delta(z_2^1 - d) \tag{22}$$

i.e. if

$$(1-\delta)(1-k(\mu)+k(0)) + \delta(d-\rho) \geq 0 \tag{23}$$

Since we must have $\rho \leq d$, we see that any μ with $1 - k(\mu) + k(0) \geq 0$ is incentive compatible. The welfare level $L = 1 - \beta - k(\mu)$ is then maximal for μ minimal, subject to $(1-\delta)\beta/\mu \le \delta\rho \le \delta d$, hence for $(1-\delta)\beta/\mu = \delta d$, and $\rho = d$. The higher is the span d, the higher will the equilibrium welfare level be.

To simplify notation, we assume in the following that $k(0) = 0$.

Consider then play under disagreement. Consider first the worst disagreement value for the worker (player 1). This is obtained when the worker exerts effort and the manager selects a monitor level μ_1 that we will now determine. So let the overall contract here call for the manager to select μ_1, the worker to exert effort, and for the parties to coordinate on continuation value $z^1 + (\rho_1, -\rho_1)$ if the monitor signal is high and μ_1 was selected; on z^1 if the monitor signal is low and μ_1 was selected; and on z^2 if the manager deviated (irrespective of the signal).

Given that μ_1 is selected, the worker will then exert effort as intended when $\delta\rho_1 \ge (1-\delta)\beta/\mu_1$, and the disagreement value that is worst for the worker will be given by

$$y^1 = (1-\delta)(-\beta, 1 - k(\mu_1)) + \delta z^1 + \delta(\rho_1, -\rho_1), \tag{24}$$

with ρ_1 minimal, thus $\rho_1 = \frac{1-\delta}{\delta}\frac{\beta}{\mu_1}$. The manager will not deviate if (similarly to (23))

$$(1-\delta)(1 - k(\mu_1)) + \delta(d - \rho_1) \ge 0 \tag{25}$$

Bargaining yields $z^1 = y^1 + \pi(L - y_1^1 - y_2^1)$ and hence

$$z^1 = (-\beta + \beta/\mu_1, 1 - k(\mu_1) - \beta/\mu_1) + \pi(L - L^1), \tag{26}$$

where $L^1 = 1 - \beta - k(\mu_1)$.

Consider next the disagreement point that is worst for player 2. This is obtained when the worker shirks and the manager selects $\mu_2 = 0$. So let the contract here call for the worker to shirk, the manager to select $\mu_2 = 0$, and for the parties to coordinate on z^2 for any outcome.[17] Nobody will then deviate, and the disagreement value will be $y^2 = (1-\delta)(0,0) + \delta z^2$. In equilibrium negotiations will prevent disagreement and lead to payoffs $z^2 = y^2 + \pi(L - y_1^2 - y_2^2)$, and this yields

$$z^2 = (0,0) + \pi(L - 0). \tag{27}$$

[17] The span is indeed maximal for $\mu_2 = 0$, as can be seen as follows. Incentives for $\mu_2 > 0$ is provided by coordination on $z^2 + (-\rho_2, \rho_2)$ if μ_2 is selected, and on z^2 if μ_2 is not selected, with $\delta\rho_2 \ge (1-\delta)k(\mu_2)$. This yields $z^2 = (-k(\mu_2), 0) + \pi(L + k(\mu_2))$, and hence a span $d = z_1^2 - z_1^1$ which is decreasing in μ_2.

It follows that the span $d = z_1^2 - z_1^1$ is given by

$$d = -(\beta/\mu_1 - \beta) + \pi_1(1 - \beta - k(\mu_1)). \tag{28}$$

The maximal span is obtained by choosing μ_1 to maximize this expression, subject to the manager's IC constraint (25) and the constraint $d \geq \rho_1 = \frac{1-\delta}{\delta}\frac{\beta}{\mu_1}$. The former constraint cannot bind, as this would imply $1 - k(\mu_1) \leq 0$. The latter may or may not bind, depending on the cost function $k(\cdot)$.

We see that the maximal span, and hence the equilibrium welfare level, will here be smaller than in the case of external enforcement of monitoring technology. (In the latter case the span was $\pi_1(1-\beta)$.) The reason is that external enforcement allows the parties to commit to a high monitor level which will be enforced in all circumstances under disagreement. Specifically, if $\mu_2 = \mu_1$ could be enforced, we would have disagreement payoff $z^2 = (0, -k(\mu_1)) + \pi(L + k(\mu_1))$ and thus span $d = -(\beta/\mu_1 - \beta) + \pi_1(1-\beta)$, which could be increased to the optimal level if in addition μ_1 could be chosen to be maximal ($\mu_1 = 1$). Absent external enforcement this is not feasible, and the welfare level will then be lower.

4.3 Options contracts: Monitoring example

Consider again the monitoring example, but suppose now that both the monitor level μ and an associated payment p from the worker can be enforced. The contract examined in Section 1 is of course a special case with zero payment. It is easily seen that a fixed payment different from zero will not affect the equilibrium outcome. But consider an externally enforceable contract with an option to choose either a high monitor level μ_1 with payment p_1, or a lower level μ_2 with payment p_2. Suppose the manager selects the option to be enforced, and that the worker observes this choice before she decides on effort. This contract is intended to be enforced under disagreement, and such that the high monitor option is selected when the worker is being punished, and the low monitor option when the worker is being rewarded.

We show here that the optimal such contract satisfies

$$p_1 - k(\mu_1) = p_2 - k(\mu_2) \tag{29}$$

with μ_1 maximal and μ_2 minimal, and that this contract strictly increases the equilibrium welfare level compared to the level that can be attained with no externally enforced payments. The reason is that the contract, when implemented as intended, affects the worker's payoffs under disagreement in such a way that the span is increased, which in turn allows for less monitoring and thus higher welfare in equilibrium.

The flexibility of the options contract (compared to a contract with a fixed payment and a fixed monitor level) is valuable here because it allows for different ways to treat the worker when she is to be rewarded, and to be punished, respectively, under disagreement. As pointed out by Bernheim and Winston (1998), such strategic flexibility–or ambiguity–can be valuable when some, but not all actions for the players are verifiable. For strategic reasons the externally enforced terms are then left incomplete to some degree, i.e. the players' actions are not restricted to the maximal extent possible.

The set of feasible games G can now be defined as follows. For a given option contract $(p_i, \mu_i)_{i=1,2}$, the stage game g has first the manager select the option to be enforced in the period. The worker observes this choice, then chooses effort privately, and finally the monitor signal is observed as before. The set G of feasible stage games encompasses all such games over the set of feasible enforceable options.

For a given option contract the analysis proceeds as in Section 1.2. Consider first the worst disagreement value for the worker (player 1). Let the overall contract here call for the manager to select (p_1, μ_1), the worker to exert effort, and for the parties to coordinate on continuation value $z^1 + (\rho, -\rho)$ if the monitor signal is high and (p_1, μ_1) was selected, and on z^1 otherwise. Given that (p_1, μ_1) is selected, the worker will then exert effort as intended when

$$\delta\rho \geq (1-\delta)\beta/\mu_1,$$

and the disagreement value that is worst for player 1 will be given by

$$y^1 = (1-\delta)(-\beta - p_1, 1 - k(\mu_1) + p_1) + \delta z^1 + \delta(\rho, -\rho), \tag{30}$$

with ρ minimal, thus $\rho = \frac{1-\delta}{\delta}\frac{\beta}{\mu_1}$. Incentive constraints for the manager's selection will be considered below.

Let $L^1 = 1 - \beta - k(\mu_1)$ be the welfare generated in the disagreement period. The parties will negotiate to avoid disagreement, and the equilibrium payoff that is worst for the worker will be $z^1 = y^1 + \pi(L - y_1^1 - y_2^1)$. A little algebra yields

$$z^1 = (-p_1 + \beta/\mu_1 - \beta, 1 - k(\mu_1) + p_1 - \beta/\mu_1) + \pi(L - L^1) \tag{31}$$

The term $\beta/\mu_1 - \beta$ is the rent accruing to the worker from his effort under imperfect monitoring.

Consider next the disagreement point that is worst for player 2. Here the contract calls for the worker to shirk, the manager to select the option (p_2, μ_2), and for the parties to coordinate on z^2 for any outcome. The manager is then willing to select the appropriate option provided $p_2 - k(\mu_2) \geq p_1 - k(\mu_1)$. This yields disagreement value

$$y^2 = (1-\delta)(-p_2, p_2 - k(\mu_2)) + \delta z^2 \tag{32}$$

In equilibrium negotiations will prevent disagreement and lead to payoffs $z^2 = y^2 + \pi(L - y_2^2 - y_2^2)$, which can now be written as

$$z^2 = (-p_2, p_2 - k(\mu_2)) + \pi(L - L^2). \tag{33}$$

Here $L^2 = -k(\mu_2)$ is the one-period welfare level should such a disagreement occur.

It follows that the span $d = z_1^2 - z_1^1$ is given by

$$d = p_1 - p_2 - (\beta/\mu_1 - \beta) + \pi_1(L^1 - L^2) \tag{34}$$

where $L^1 - L^2 = 1 - \beta - k(\mu_1) + k(\mu_2)$, and IC for the manager's selection of (p_2, μ_2) requires $k(\mu_1) - k(\mu_2) \geq p_1 - p_2$. We see that the span is maximal when $p_1 - p_2$ is maximal (and thus when (29) holds), and consequently when μ_1 is maximal and μ_2 minimal ($\mu_1 = 1$, $\mu_2 = 0$). The maximal span is thus

$$d = (k(1) - k(0))(1 - \pi_1) + \pi_1(1 - \beta)$$

The options contract allows the parties to adjust the monitor level under disagreement such that the level is inefficiently high only in the case where shirking is to be avoided. This results in a larger span than what is obtained without the option contract, where the monitor level is inefficiently high in all cases under disagreement, and the span is $\pi_1(1-\beta)$. The latter situation implies a larger welfare difference $L^1 - L^2$, which in isolation yields a larger span, but this is more than compensated for in the options contract via the payment difference $p_1 - p_2$.

While the long term option contract specifies either maximal or no monitoring, in equilibrium the parties agree each period to an intermediate monitor level μ, which is the minimal level necessary to induce effort from the worker, and thus given by $\frac{1-\delta}{\delta}\frac{\beta}{\mu} = d$. The larger span allows for a lower equilibrium μ and thereby a higher level of welfare.

It remains to verify that the manager has no incentives to deviate from selecting the option (p_1, μ_1) when the worker is to be punished. Note that complience gives payoff $y_2^1 = (1-\delta)(1-k(\mu_1)+p_1-\beta/\mu_1)+\delta z_2^1$. A deviation to (p_2, μ_2) with $\mu_2 = 0$ would make the worker shirk, and the manager's payoff would then be $(1-\delta)(-k(\mu_2)+p_2)+\delta z_2^1$. The latter is smaller than y_2^1 due to (29) and $\beta/\mu_1 = \beta < 1$. This verifies that all IC constraints are satisfied.[18]

Decision rights

The right to select an option can in principle be contracted on, and be externally enforced. Above we have assumed that this right belongs to the manager. Assume now that the worker has the right.[19] Consider again options of the form (p_i, μ_i), $i = 1, 2$, where (p_i, μ_i) is intended to be selected under disagreement to punish player i, and p_i is a payment from the worker to the manager.

Consider first the disagreement point that is worst for the manger (player 2). The contract here calls for the worker to selct the option (p_2, μ_2), then to shirk, and the parties to coordinate on z^2 for any outcome. These actions for the worker are incentive compatible if $p_2 \leq p_1$, and the payoffs are then as above given by (32) and (33) under disagreement and agreement, respectively.

Next consider the disagreement point that is worst for player 1, where it is intended that the worker selects (p_1, μ_1) and exerts effort $a = 1$. Let the contract here call for coordination on $z^1 + (\rho, -\rho)$ if the worker selects (p_1, μ_1) and the signal is high, and on z^1 otherwise, where as above $\rho = \frac{1-\delta}{\delta}\frac{\beta}{\mu_1} \leq d$. For the given option, effort is then incentive compatible and leads as above to disagreement values given by (30).

If the worker deviates and selects the other option (p_2, μ_2), he will optimally shirk, and thus get payoff $(1-\delta)(-p_2) + \delta z_1^1$. Option (p_1, μ_1) is thus his best choice if $\delta\rho \geq (1-\delta)(p_1 - \beta - p_2)$, i.e. if $\beta(1/\mu_1 - 1) \geq p_1 - p_2$.

Negotiations then yield values z^1 as in (31), and consequently a span given by (34) above. But now the IC constraints for the worker's selection of options are $\beta(1/\mu_1 - 1) \geq p_1 - p_2 \geq 0$, and we must moreover have $\rho = \frac{1-\delta}{\delta}\frac{\beta}{\mu_1} \leq d$. From (34) we see that the largest span is obtained when $p_1 - p_2$ is maximal, and thus here $d = \pi_1(L^1 - L^2) = \pi_1(1 - \beta - k(\mu_1) + k(\mu_2))$. This expression being increasing in μ_2 and decreasing in μ_1

[18]It is straightforward to verify that participation constraints for the worker $(y_1^1, y_1^2 \geq 0)$ can be satisfied by e.g. setting p_1 such that $y_1^1 = 0$ and $p_2 = p_1 - k(1) + k(0)$.

[19]For example, a traveling salesman or service worker may have the right to control the extent to which his movements are registered by e.g. GPS.

implies that the span will be maximal when $\mu_2 = 1$ and μ_1 is the smallest solution to

$$\frac{1-\delta}{\delta}\frac{\beta}{\mu_1} = d = \pi_1(1-\beta-k(\mu_1)+k(1)) \tag{35}$$

We also see that this can be implemented without violating participation constraints for the worker by e.g. setting $p_2 = 0$ and $p_1 = \beta\frac{1-\mu_1}{\mu_1}$.

Finally, to implement effort under agreement, the monitor level μ must satisfy $\frac{1-\delta}{\delta}\frac{\beta}{\mu} \leq d$, where now d is given by equation (35). This implies $\mu = \mu_1$, i.e. that the monitor levels under agreement and under disagreement to punish the worker should be equal. Consequently, in this case of worker control, the externally enforced terms in the inherited contract need not be renegotiated in the current period. Keeping the option menu fixed, the worker has incentives to select the appropriate μ_i under disagreement, and the parties will agree on $\mu = \mu_1$ otherwise.

The externally enforced part of the contract in this case can be seen as specifying a ”normal” level of monitoring to be applied whenever the worker is supposed to provide effort, and a very high and inefficient level ($\mu_2 = 1$) when it is intended that she shirks (under disagreement). This arrangement, including the inefficient monitor level in conjunction with shirking, induces the largest feasible span when the worker controls the selection of options. Recall that in the case of manager control considered above, the largest feasible span is obtained with inefficiently high monitoring when the worker is supposed to provide effort under disagreement. The differences between the two cases reflect the differences in the two parties' incentives when choosing between options. Under worker control, incentive constraints for the selection of options imply that the span is proportional to the welfare difference $L^1 - L^2$; and thus largest when the monitor level μ_2 is highly inefficient.[20]

Comparing with the case of manager control, where the span is $(1-\pi_1)(k(1)-k(0)) + \pi_1(1-\beta)$, we see that manager control is better (allows for a lower μ under agreement) when $\pi_2(k(1)-k(0)) > \pi_1(k(1)-k(\mu_1))$. Manager control is thus better only if the manager's bargaining strength is not too weak. But if it is weak, worker control is strictly better. This illustrates that allocation of decision rights can be important in this setting.

4.4 Multitasking

Consider a principal-agent relation where an agent supplies efforts on two tasks with, respectively verifiable and non-verifiable but observable outputs. For concreteness we refer

[20] The level of inefficiency may be limited by constraints neglected here, e.g. participation constraints for the manager. This can be straightforwardly included in the analysis.

to these as quantity (a_2) and quality (a_1), respectively. The good belongs to the principal. Payments conditional on quantities of the good can now be externally (court) enforced. The agent has costs $\kappa(a_1, a_2)$, and the principal's gross value is $p(a_1, a_2)$. Suppose quality is binary; either high ($a_1 = a_{1h}$) or low ($a_1 = 0$), and that a high quality good is more costly for the agent and more valuable for the principal, so $p(a_{1h}, a_2) - \kappa(a_{1h}, a_2) > p(0, a_2) - \kappa(0, a_2) \geq 0$ for all quantities $a_2 \in (0, \bar{a}_2)$. Assume also that $p(a_1, 0) = \kappa(a_1, 0) = 0$ for both qualities.

The interesting problem is to implement high quality from the agent, since the optimal quantity of the low-quality good can be implemented by an externally enforced contract. The contractual equilibrium will have internally enforced and externally enforced terms, and, as we know, it will be semi-stationary.

At the beginning of a period, the externally enforced terms for the current period may be renegotiated. Suppose the parties agree to externally enforced payment and quantity (p, a_2). Let z^A and z^P be the endpoints of the equilibrium payoff set, where z^A is worst for the agent, and z^P is worst for the principal. High quality from the agent is then incentive compatible if $(1 - \delta)(p - \kappa(a_{1h}, a_2)) + \delta z^P_A \geq (1 - \delta)(p - \kappa(0, a_2)) + \delta z^A_A$, i.e. if

$$\kappa(a_{1h}, a_2) - \kappa(0, a_2) \leq \frac{\delta}{1 - \delta}(z^P_A - z^A_A) \tag{36}$$

The welfare level attained is $L = p(a_{1h}, a_2) - \kappa(a_{1h}, a_2)$. The condition makes clear that internally enforced incentives, represented by the span $z^P_A - z^A_A$, must be sufficiently large to make up for the agent's additional cost to provide high rather than low quality; and that the higher the span, the higher is the welfare level that can be sustained.

We will show that, under some conditions, the largest equilibrium span can be obtained by letting the externally enforced terms take the form of a payment schedule that exactly compensates the agent for the costs to produce low quality, conditional on the verifiable quantity provided. Moreover, these terms need not be renegotiated in equilibrium in any period, so the externally enforced terms of the overall contract can be taken to be fully stationary in such a case.

In this payment scheme, the agent can be seen as having the right to choose a quantity-payment pair from the schedule under agreement, and most importantly, under disagreement. While this ensures the largest span under some conditions, it turns out that under other conditions, it is better to let the principal have the right to select quantities and payments, i.e. let the externally enforced terms take the form of an options contract for the principal. We show that incentive compatible selection to ensure a largest span can then be obtained by options were the court-enforced payments equal the principal's low-quality value for the selected quantities.

We turn now to the analysis. The externally enforced terms of the overall contract are important for play under disagreement, and as seen in previous examples, it can be advantageous to allow some flexibility and let one of the players have the right to choose from a set of options. Since courts can enforce payments conditional on verifiable quantities, the feasible options are here a set of quantity-payment pairs. Two options are sufficient for disagreement play, specifying quantity-payment pairs (a_2^i, p^i), $i = A, P$ intended to be selected when punishing the agent and punishing the principal, respectively. The payments are from the principal to the agent.

Consider first the case where the agent has the right to select from the options. He can do so by selecting the verifiable quantity a_2^i, the associated payment will then be enforced by the court.

Under disagreement to punish the principal, let the contract call for the agent to supply $a = (0, a_2^P)$, and the parties to coordinate on z^P next period for every outcome this period. This is incentive compatible for the agent as long as $p^P - \kappa(0, a_2^P) \geq p^A - \kappa(0, a_2^A)$, and leads to disagreement values

$$y^P = (1-\delta)(p^P - \kappa(0, a_2^P), p(0, a_2^P) - p^P) + \delta z^P \tag{37}$$

Under disagreement to punish the agent, let the contract call for the agent to supply $a = (a_{1h}, a_2^A)$, and the parties to coordinate on $z^A + (\rho, -\rho)$ unless the agent deviates, in which case they coordinate on z^A. For given quantity a_2^A, high quality is incentive compatible for the agent if

$$(1-\delta)(p^A - \kappa(a_{1h}, a_2^A)) + \delta(z_A^A + \rho) \geq (1-\delta)(p^A - \kappa(0, a_2^A)) + \delta z_A^A$$

with $\rho \leq d = z_A^P - z_A^A$. To maximally punish the agent, ρ should be minimal and thus given by

$$(\kappa(a_{1h}, a_2^A) - \kappa(0, a_2^A))(1-\delta) = \delta\rho \tag{38}$$

Selecting quantity a_2^A is then incentive compatible for the agent if $p^A - \kappa(0, a_2^A) \geq p^P - \kappa(0, a_2^P)$. This leads to disagreement values

$$y^A = (1-\delta)(p^A - \kappa(0, a_2^A)), p(a_{1h}, a_2^A) - \kappa(a_{1h}, a_2^A) - (p^A - \kappa(0, a_2^A))) + \delta z^A \tag{39}$$

Negotiations yield $z^i = y^i + \pi(L - y_A^i - y_P^i)$, and thus

$$\begin{aligned} z_A^P &= (p^P - \kappa(0, a_2^P)) + \pi_A(L - (p(0, a_2^P) - \kappa(0, a_2^P)) \\ z_A^A &= (p^A - \kappa(0, a_2^A)) + \pi_A(L - (p(a_{1h}, a_2^A) - \kappa(a_{1h}, a_2^A)) \end{aligned}$$

Recall that incentive compatibility now requires $p^A - \kappa(0, a_2^A) = p^P - \kappa(0, a_2^P)$, hence we see that the span $d = z_A^P - z_A^A$ is largest for $a_2^P = 0$, and then given by

$$d = \pi_A(p(a_{1h}, a_2^A) - \kappa(a_{1h}, a_2^A))$$

This implies $p^A - p^P = \kappa(0, a_2^A)$, and we can set $p^P = \kappa(0, 0) = 0$. Thus, the court enforced payment compensates the agent for the cost of providing low quality of the selected quantity. Moreover, the optimal a_2^A is the maximal quantity (of the high quality good) that can be implemented with the equilibrium span, thus it coincides with the equilibrium quantity a_2 under agreement. Hence it is given as the largest solution to

$$\kappa(a_{1h}, a_2) - \kappa(0, a_2) = \frac{\delta}{1-\delta}\pi_A(p(a_{1h}, a_2) - \kappa(a_{1h}, a_2))$$

Note that the externally enforced terms can here be specified as a payment schedule $p(a_2') = \kappa(0, a_2')$, for any $a_2' \geq 0$, i.e. specifying that the agent is paid the cost of supplying the low quality good. Under the overall contract, it is then incentive compatible for the agent to provide $a = (a_{1h}, a_2)$ both under agreement and under disagreement to reward the principal, and to provide $a = (0, 0)$ under disagreement to punish the principal. The externally enforced terms need thus not be renegotiated under agreement in this case.

It is of interest to compare this setting to an environment with no external enforcement. It follows from Miller-Watson (2013) that the contractual equilibrium under no external enforcement is to provide the high-quality good in a quantity given as the largest solution to

$$\kappa(a_{1h}, a_2) = \frac{\delta}{1-\delta}\pi_A(p(a_{1h}, a_2) - \kappa(a_{1h}, a_2))$$

External enforcement improves welfare by allowing a larger quantity of the high quality good to be sustained in equilibrium.

The payment schedule allows the agent to select the externally enforced terms under disagreement. As noted above, this right may alternatively be allocated to the principal, giving her the right to select from two options (a_2^i, p^i), $i = A, P$. Assuming courts can enforce specific performance, the agent must comply with the specified quantity when the principal selects (a_2^i, p^i). [21]

So assume now that the principal has the right to selcet among the options. Under disagreement to punish the principal (reward the agent), let the overall contract call for the principal to select option (a_2^P, p^P), the agent to provide $a = (0, a_2^P)$, and for the parties continue with z^P next period for any outcome this period. This is incentive compatible provided $p(0, a_2^P) - p^P \geq p(0, a_2^A) - p^A$ with $p^P \geq \kappa(0, a_2^P)$, and leads to disagreement values as in (37) above.

Under disagreement to punish the agent, let the contract call for the principal to select option (a_2^A, p^A) and the agent to provide $a = (a_{1h}, a_2^A)$. Let the contract also call for coordination on z^A if only the agent deviates, and on $z^A + (\rho, -\rho)$ otherwise, where $\rho \geq 0$ is given by (38).

Given the option (a_2^A, p^A), high quality is then incentive compatible for the agent. We must of course have $\rho \leq d$, and thus $a_2^A \leq a_2$ (the quantity supplied under agreement). Note that by setting $\rho = 0$, we may allow $a_2^A = 0$.

The principal's choice of option is incentive compatible if $p(a_{1h}, a_2^A) - p^A \geq p(0, a_2^P) - p^P$. (If she deviates, the agent will supply low quality, and the principal will then be worse off.) Substituting for ρ, we then see that disagreement values are here given as in (39) above.

By internal bargaining consistency this leads to the same expressions as above for the values z_A^A, z_A^P, but incentive compatibility for the principal now requires

$$p(a_{1h}, a_2^A) - p^A \geq p(0, a_2^P) - p^P \geq p(0, a_2^A) - p^A$$

The span $d = z_A^P - z_A^A$ is then largest for payments such that $p^P - p^A = p(0, a_2^P) - p(0, a_2^A)$, which yields

$$\begin{aligned} d &= (1 - \pi_A)(p(0, a_2^P) - \kappa(0, a_2^P)) \\ &\quad + \pi_A(p(a_{1h}, a_2^A) - \kappa(a_{1h}, a_2^A)) - (p(0, a_2^A) - \kappa(0, a_2^A)) \end{aligned}$$

The maximal span with these options is obtained by choosing $a_2^P = \arg\max_{a_2}(p(0, a_2) -$

[21] If specific performance is not enforced, we may assume that deviations from the specified quantity are discouraged by sufficiently low associated payments, enforced by the court.

$\kappa(0, a_2)) \equiv a_2^0$, and a_2^A to maximize the expression in the last line, hence we have

$$\begin{aligned} d &= d(a_2) = (1-\pi_A)\left(p(0,a_2^0) - \kappa(0,a_2^0)\right) \\ &\quad + \max_{0 \le a_2' \le a_2} \left[\pi_A(p(a_{1h}, a_2') - \kappa(a_{1h}, a_2')) - (p(0,a_2') - \kappa(0,a_2'))\right] \end{aligned}$$

The incentive compatible payments can be set as $p^i = p(0, a_2^i)$, $i = A, P$, implying that the agent is paid the gross value of low quality in this scheme.

The equilibrium quantity of the (high quality) good is now given by the largest solution to [22]

$$\kappa(a_{1h}, a_2) - \kappa(0, a_2) = \frac{\delta}{1-\delta} d(a_2).$$

Comparing the two schemes considered above, we see that the payment schedule, where the agent selects the externally enforced terms to be applied under disagreement, dominates for π_A sufficiently large. In this scheme incentive compatible selection is obtained by court-enforced payments that compensate the agent for the costs of providing low quality. The options scheme, where the principal controls this selection, dominates for π_A sufficiently small. In this scheme, incentive compatible selection is obtained by the court enforcing payments equal to the value of providing low quality. The comparison demonstrates that allocation of decision rights can be an important element of the equilibrium contract.

A Recursive Characterization

In this appendix, we perform analysis that yields a recursive characterization of contractual equilibrium payoffs, along the lines of Abreu, Pearce, and Stacchetti (1990) and Miller and Watson (2013), where one relates continuation values that can be achieved from a given period to the continuation values in the next period. The key complication we face here is that the set of continuation values generally differs across periods and must be indexed by the inherited external contract. Thus, instead of looking for a fixed point set of continuation values, as is the case in the earlier literature, we are looking for a fixed point in the space of *collections of* sets of continuation values.

[22] It may be noted that, if the maximal span $d(a_2)$ is obtained for $a_2' = a_2$, then the externally enforced option contract need not be renegotiated in any period, since it will implement high-quality quantity a_2 also in agreement. Otherwise it will be renegotiated under agreement.

A.1 Self-Generation

To get into the details, let us start by describing continuation values that can be achieved from the action phase in a given period t. For the following definition, take as given any collection $\mathcal{W} = \{W(c')\}_{c' \in C}$, where $W(c') \subset \mathbb{R}^2$ for every $c' \in C$. This collection describes the continuation values that can be selected from the start of period $t+1$, as a function of the external contract inherited in period $t+1$.

Definition 11. *Take any $c \in C$ and let $g(c) = (A, X, \lambda, u, P)$ be the stage game designated for the current period under external contract c. Say that w is* **c-supported** *relative to $\mathcal{W}$ if there exists a mixed action profile $\alpha \in \Delta A$ and a function $z\colon X \times [0,1] \to \mathbb{R}^2$, such that*

- *for all $x \in X$ and $\phi \in [0,1]$, it is the case that $z(x,\phi) \in W(\zeta(c,x,\phi))$;*
- *α is a Nash equilibrium of $\langle A, E_{x,\phi}\left[(1-\delta)u(\cdot,x) + \delta z(x,\phi)\right]\rangle$, where for each $a \in A$ the expectation is taken with respect to $x \sim \lambda(a)$ and $\phi \sim U[0,1]$; and*
- *the expected payoff of this Nash equilibrium is w.*

In reference to the first two conditions, we say that α is **c-enforced** *relative to $\mathcal{W}$.*

Suppose that the players enter a period with c as their inherited external contract, suppose a particular continuation value $\underline{w}$ is c-supported, and assume that the players would coordinate to achieve $\underline{w}$ in the event that they fail to reach an agreement in the negotiation phase. Further, imagine that the players are able to achieve a joint value of L through negotiation. Then, incorporating the bargaining solution, the continuation value will be $w = \underline{w} + \pi(L - \underline{w}_1 - \underline{w}_2)$ from the beginning of the period.[23]

For a given collection $\mathcal{W} = \{W(c')\}_{c' \in C}$, any $c \in C$, and any level L, let $B^L(c,\mathcal{W})$ be the set of continuation values that can be achieved from the beginning of a period in which c is the inherited contract and the players bargain to obtain level L:

$$B^L(c,\mathcal{W}) = \{\underline{w} + \pi(L - \underline{w}_1 - \underline{w}_2) \mid \underline{w} \text{ is } c\text{-supported relative to } \mathcal{W}\}. \tag{40}$$

Then, if $\mathcal{W}$ is the collection of continuation values available from the start of some period $t+1$, we know that $\mathcal{W}' = \{B^L(c',\mathcal{W})\}_{c' \in C}$ is the set of continuation values attainable from the start of period t.

Let us apply operator B to characterize the continuation values for a single regime r. For every $c \in C$, let $V(c;r)$ be the set of continuation values for regime r following

[23]We can ignore whether $L \geq \underline{w}_1 - \underline{w}_2$ for now, realizing that in a contractual equilibrium this inequality will hold (nonnegative surplus) after every history.

histories in which c is the inherited external contract. That is,

$$V(c;r) = \{v(h;r) \mid h \in H \text{ and } \hat{c}(h) = c\},$$

and note that we have $V(c;r) = \emptyset$ if there is no history h for which the inherited contract $\hat{c}(h)$ is c. Let us define $\mathcal{V}(r) = \{V(c';r)\}_{c' \in C}$.

Lemma 3. *If regime r is incentive compatible in the action phase and internally bargain-consistent, then*

- *there is an external contract $\overline{c} \in C$ and a continuation value w that is $\overline{c}$-supported relative to $\mathcal{V}(r)$ and satisfies $L = w_1 + w_2$, where L is the regime's level; and*
- *$V(c;r) \subset B^L(c, \mathcal{V}(r)) \neq \emptyset$ for all $c \in C$.*

Proof of Lemma 3. Take any history to the action phase of a period, (h, c, m). From this history, the regime r prescribes individual actions that are incentive compatible with respect to continuation values that the regime supports. Thus, for every $c \in C$, there is a continuation value from the action phase $\underline{w}$ that is c-supported relative to $\mathcal{W}$. This implies that $B^L(c, \mathcal{V}(r)) \neq \emptyset$. Internal bargain-consistency at the regime's level L thus implies $V(c;r) \subset B^L(c, \mathcal{V}(r))$. The first claim pertains to the manner in which level L is attained. Regime r specifies that the players jointly select an external contract and coordinate on individual actions that achieve level L, which the first claim. □

As one would expect, the reverse implication also holds.

Definition 12. *Consider a collection $\mathcal{W} = \{W(c')\}_{c' \in C}$. We say that $\mathcal{W}$ is* **self-generating with level L** *if there is an external contract $\overline{c} \in C$ and a $\overline{c}$-supported continuation value w that satisfies $L = w_1 + w_2$, and if*

$$W(c) \subset B^L(c, \mathcal{W}) \text{ for all } c \in C. \tag{41}$$

Call $\mathcal{W}$ **fully nonempty** *if $W(c) \neq \emptyset$ for all $c \in C$.*

Lemma 4. *If a collection $\mathcal{W}$ is fully nonempty and self-generating with level L, then there is a regime r that is incentive compatible in the action phase and internally bargain-consistent, has level L, and has the property that $V(c;r) \subset W(c)$ for all $c \in C$.*

Proof of Lemma 4. The result follows from standard arguments. We construct the regime r by, for every history, specifying the behavior identified in the self-generation conditions.

Start with the null history, h^0, and pick any element $w \in W(\underline{c})$ to be the equilibrium continuation value from the beginning of the game. From the self-generation conditions, we can find a $\underline{c}$-supported disagreement value $\underline{w} \in B^L(\underline{c}, \mathcal{W})$ such that $w = \underline{w} + \pi(L - \underline{w}_1 - \underline{w}_2)$. Also, there exists an external contract $\overline{c}$ such that some continuation value with joint value L is $\overline{c}$-supported.

Prescribe $r^{\mathrm{c}}(h^0) = \overline{c}$ and let $r^{\mathrm{m}}(h^0)$ to be the corresponding transfer that achieves w as the continuation value. Then prescribe $r^{\mathrm{a}}(h, \underline{c}, 0)$ to be the mixed action α that is identified by self-generation to $\underline{c}$-support $\underline{w}$. Likewise, prescribe $r^{\mathrm{a}}(h, r^{\mathrm{c}}(h^0), r^{\mathrm{m}}(h^0))$ to be the mixed action identified to $\underline{c}$-support $\overline{w}$. For other values of (c^1, m^1), the prescribed action profile $r^{\mathrm{a}}(h, c^1, m^1)$ is that identified to c^1-support an arbitrary continuation value from the action phase, among those that can be c^1-supported. There must be such an element, for otherwise $B^L(c^1, \mathcal{W})$, and hence $W(c^1)$, would be empty (contradicting that $\mathcal{W}$ is fully nonempty).

The construction continues by looking at all of the histories reaching period 2. For each such history h, a specific continuation value from $W(\hat{c}(h))$ is required to provide the incentives and continuation payoffs specified in period 1. We simply repeat the steps in the previous paragraph to specify behavior in period 2 following history h. The process continues for period 3, 4, and so on, which inductively yields a fully specified regime. By construction from the self-generation conditions, the regime's continuation values have the desired properties and the regime is incentive compatible in the action phase and internally bargain-consistent. $\square$

A.2 Computation Procedure and Existence

To review the analysis thus far, self-generation relates to the conditions of incentive compatibility in the action phase and internal bargain-consistency for a regime. To characterize contractual equilibrium, we must look across all regimes with these properties and find one with the highest level (if it exists). Fortunately, we can compare regimes with different levels by normalizing to level zero. To this end, for a vector $\eta \in \mathbb{R}^2$ and a collection of sets $\mathcal{W} = \{W(c)\}_{c \in C}$, let "$\mathcal{W}+\eta$" denote the collection that results by adding η to all the points in the sets. That is, $\mathcal{W} + \eta = \{W(c) + \eta\}_{c \in C}$, where $W(c) + \eta = \{w + \eta \mid w \in W(c)\}$.

Note that the incentive conditions in Definition 11 would not be affected by transforming all of the sets in $\mathcal{W}$ by a constant vector; such a transformation merely adds a constant to z, which yields a strategically equivalent induced game $\langle A, E\left[(1-\delta)u(\cdot, x) + \delta z(x, \phi)\right]\rangle$. As a result, the c-supported continuation values are all transformed by the same constant times δ. We also see that, from the definition of $B^L(c, \mathcal{W})$ in Equation 40, changing the value of L causes the set $B^L(c, \mathcal{W})$ to merely shift by a constant multiple of the vector π.

These facts imply that a collection $\mathcal{W}$ satisfies $W(c) \subset B^L(c, \mathcal{W})$ for all $c \in C$ if and only if the collection $\mathcal{W}^0 = \{W^0(c)\}_{c \in C} \equiv \mathcal{W} - L\pi$ satisfies $W^0(c) \subset B^0(c, \mathcal{W}^0)$ for all $c \in C$. So, we can replace self-generation Condition 41 in Definition 12 with the normalized version where $L = 0$ and the collection is $\mathcal{W}^0$. When we are dealing with such a normalized collection $\mathcal{W}^0$, let us say that it is **normalized self generating** if it satisfies $W^0(c) \subset B^0(c, \mathcal{W}^0)$ for all $c \in C$.

Let $\mathcal{W}^* = \{W^*(c)\}_{c \in C}$ be the union of all normalized self-generating collections of continuation-value sets. That is, for every $c \in C$, $W^*(c)$ is the union of the sets $W^0(c)$ across all normalized self-generating collections $\mathcal{W}^0 = \{W^0(c)\}_{c \in C}$. It is clear that operator B^0 is monotone and the set of c-enforced action profiles is increasing in $\mathcal{W}^0$. Thus, $\mathcal{W}^*$ is normalized self-generating and contains every other normalized self-generating collection. Because the contractual equilibria maximize the level, we can determine existence and find the contractual-equilibrium level by examining this union collection. To be precise, the foregoing analysis proves the next result. In this theorem, for every $c \in C$, $u(\cdot; c)$ denotes the payoff function of stage game $g(c)$ and $\lambda(\cdot; c)$ denotes the outcome distribution function for this stage game.

Theorem 3. *For any relational-contract setting, a contractual equilibrium exists if and only if $\mathcal{W}^*$ is fully nonempty and if the following optimization problem has a solution:*

$$\begin{aligned} \max_{\substack{c \in C \\ \alpha \in \Delta A(c)}} \quad & E_{a,x}\left[u_1(a, x; c) + u_2(a, x; c) \mid x \sim \lambda(a; c), a \sim \alpha\right] \\ \textit{subject to:} \quad & \alpha \text{ is } c\text{-enforced relative to } \mathcal{W}^*. \end{aligned} \tag{42}$$

The contractual-equilibrium level L^* is the maximized value of this optimization problem.

In the applications we present elsewhere in this paper, our analysis follows Optimization Problem 42. Another route to existence is to assume enough finiteness:

Theorem 4. *For any relational-contract setting in which C is finite and every game in G is finite, a contractual equilibrium exists.*

Proof of Lemma 4. We start by proving that there is a collection of continuation-value sets $\mathcal{W}^0$ that satisfies $W^0(c) \subset B^0(c, \mathcal{W}^0)$ for all $c \in C$. In particular, we will work with collections that have singleton continuation-value sets: for each $c \in C$, $W^0(c) = \{w^c\}$, where $w^c \in \mathbb{R}^2$. Note that $w_1^c + w_2^c = 0$ for all $c \in C$, so we can think of these points as being on the real line.

For any point $y = (w^c)_{c\in C} \in \mathbb{R}^{|C|}$ that defines $\mathcal{W}^0$ by $W^0(c) = \{w^c\}$ for all $c \in C$, let $f(y) \equiv \prod_{c\in c} \operatorname{Conv} B^0(c, \mathcal{W}^0)$, where "Conv" denotes the convex hull. Because the stage games are finite, the bargaining solution maps supported values to the zero-value line along the ray π, and continuation values are discounted, we can find a bound η such that $w^c \in [-\eta, \eta]^2$ for all $c \in C$ implies that $B^0(c, \mathcal{W}^0) \in [-\eta, \eta]^2$. Further, because each stage game is finite and the Nash correspondence is nonempty and upper hemi-continuous in payoff vectors, B^0 has the same property. Thus, f is a correspondence from a compact set to itself, it is nonempty and convex valued, and it is upper-hemicontinuous. By the Kakunati fixed-point theorem, f has a fixed point $\overline{y} = (\overline{w}^c)_{c\in C}$.

Let $\overline{\mathcal{W}} = \{\overline{W}(c)\}_{c\in C}$ be defined by $\overline{W}(c) = \{\overline{w}^c\}$ for all $c \in C$. The fixed point property means that $\overline{W}(c) \subset \operatorname{Conv} B^0(c, \overline{\mathcal{W}})$ for all $c \in C$, but it is not necessarily the case that $\overline{W}(c) \subset B^0(c, \overline{\mathcal{W}})$ for all $c \in C$. However, if this latter condition fails, then we can find two points $\overline{w}'^c, \overline{w}''^c \in B^0(c, \overline{\mathcal{W}})$ such that $\overline{w}^c$ is on the line between $\overline{w}'^c$ and $\overline{w}''^c$. The players can achieve an expected continuation value of $\overline{w}^c$ in a period in which the inherited external contract is c, from the perspective of the previous period by using the random draw ϕ to randomize between $\overline{w}'^c$ and $\overline{w}''^c$.[24] For each affected external contract $c \in C$, we replace $\overline{W}(c) = \{\overline{w}^c\}$ with $\overline{W}(c) = \{\overline{w}'^c, \overline{w}''^c\}$, and the adjusted $\overline{\mathcal{W}}$ satisfies $\overline{W}(c) \subset B^0(c, \overline{\mathcal{W}})$ for all $c \in C$. Because $\overline{\mathcal{W}}$ is fully nonempty, continuation values from the action phase are supported (for every external contract) and there is a level L such that $\overline{\mathcal{W}} + L\pi$ is self-generating with level L. This implies that $\mathcal{W}^*$ is fully nonempty.

To complete the proof, we must show that Optimization Problem 42 has a solution. By upper hemi-continuity of B^0 and that $\mathcal{W}^*$ is normalized self-generating, we know that the closure of $\mathcal{W}^*$, denoted by $\operatorname{Clos} \mathcal{W}^*$, is also normalized self-generating. Here, $\operatorname{Clos} \mathcal{W}^* = \{\operatorname{Clos} W^*(c)\}_{c\in C}$. Thus, for each $c \in C$, the problem of maximizing $u_1(\alpha; c) + u_2(\alpha; c)$ over all c-enforced action profiles $\alpha \in \Delta A(c)$ has a solution. Because there are a finite number of external contracts, the overall maximum exists. □

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[24] Because ϕ is uniformly distributed, for any outcome that would lead to external contract c for a positive mass of the random draw, the players can divide this set of ϕ values to achieve any probability distribution over the continuation values $\overline{w}'^c$ and $\overline{w}''^c$.

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